Lecture 10

2025/2026

Microwave Devices and Circuits for Radiocommunications

2025/2026

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday **12-14**, **P2**
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 24.02.2026 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

2025/2026

- Laboratory – **associate professor Radu Damian**
  - Monday 14-16, II.13 / (even weeks)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 24.02.2026)
    - personal homework

General theory

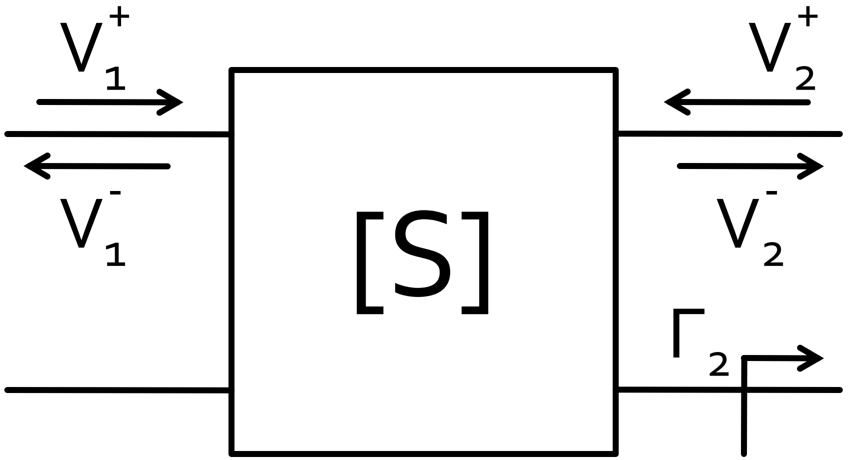
Microwave Network Analysis

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

Scattering matrix – S

- Scattering parameters



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Power waves for N ports

$$[b]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}\cdot[a]$$

- The scattering matrix for power waves, $[S_p]$

$$[b]=[S_p]\cdot[a]$$

$$[S_p]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}$$

- But: $[S]=\left([Z]-[Z_0]\right)\cdot\left([Z]+[Z_0]\right)^{-1}$

- Typically

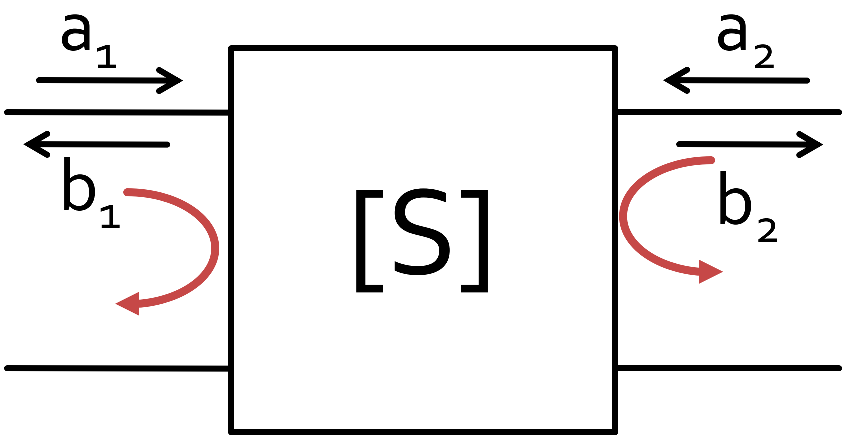
$$Z_{0i}=Z_{Ri}=R_0,\forall i$$

$$R_0=50\Omega$$

$$[S_p]\equiv[S]$$

- **they coincide!!!**

Scattering matrix – S

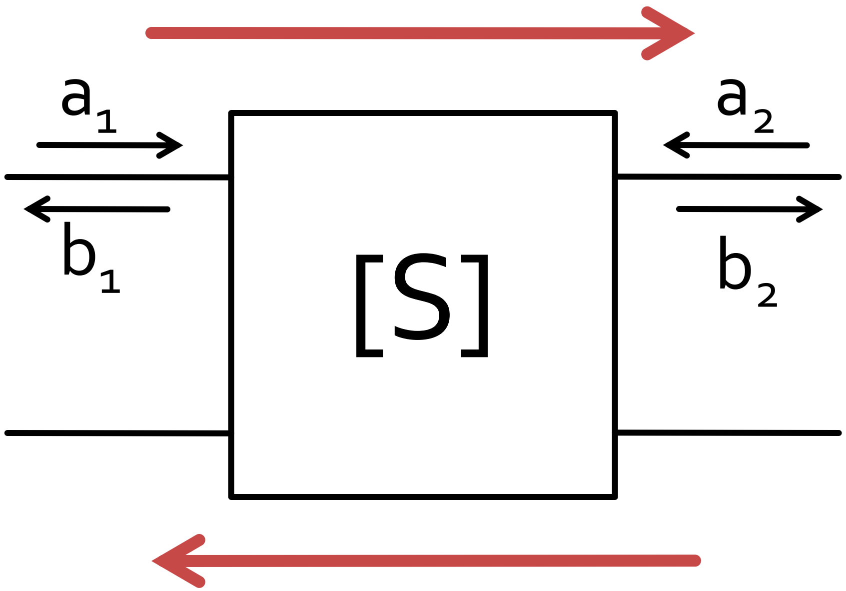


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

- S_{11} and S_{22} are **reflection coefficients** at ports 1 and 2 when the other port is **matched**

Scattering matrix – S

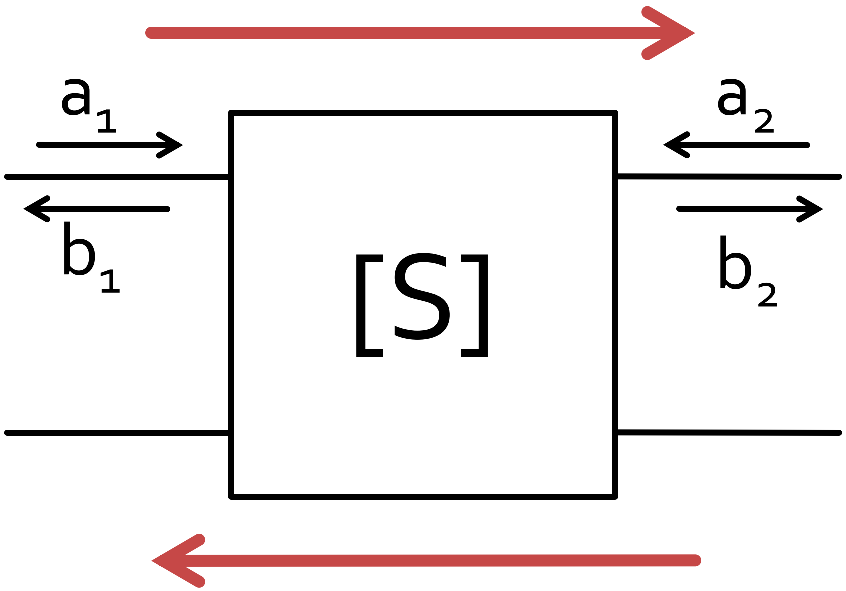


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \qquad S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

- S_{21} si S_{12} are signal amplitude **gain** when the other port is **matched**

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

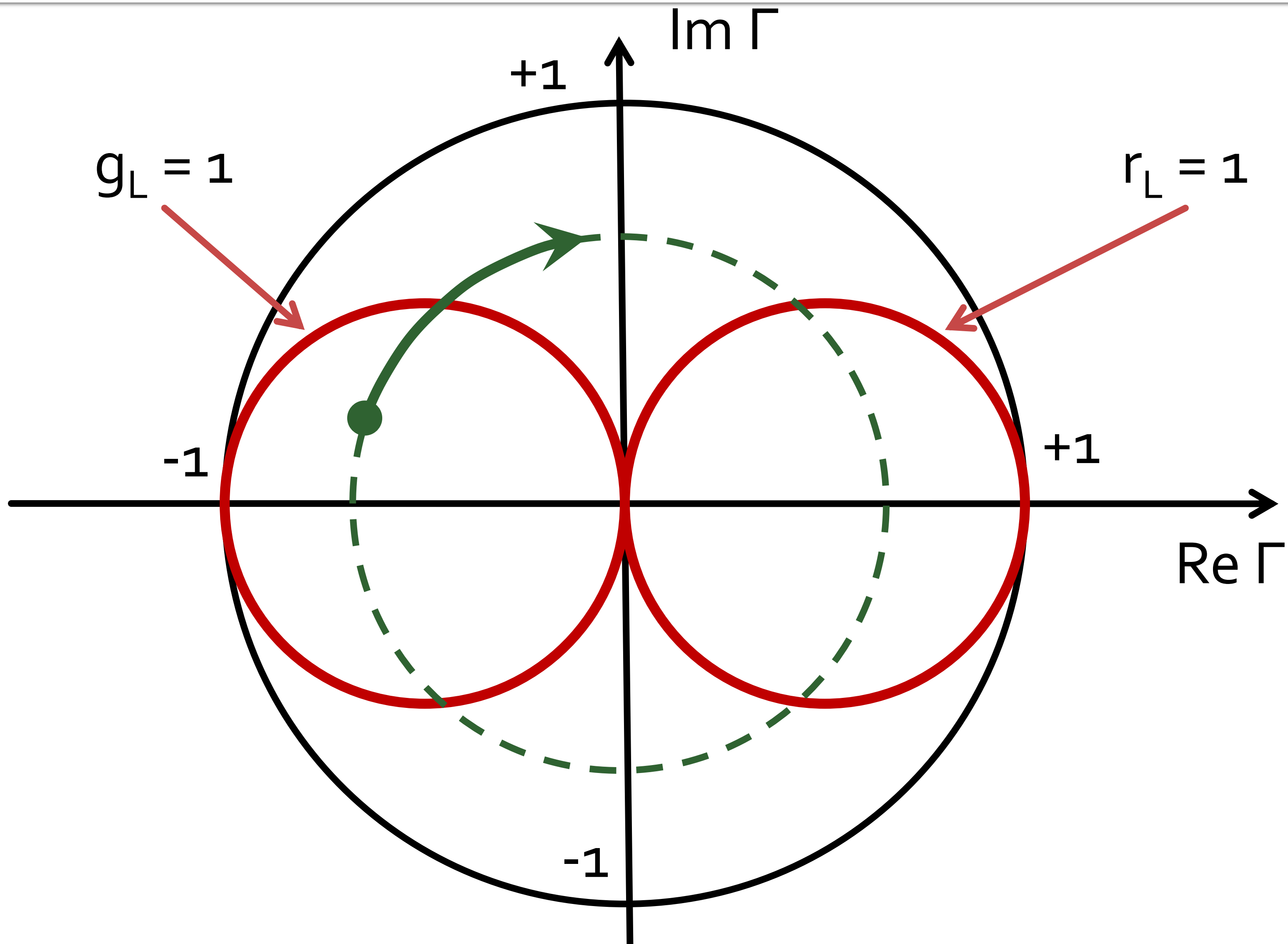
$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Impedance Matching

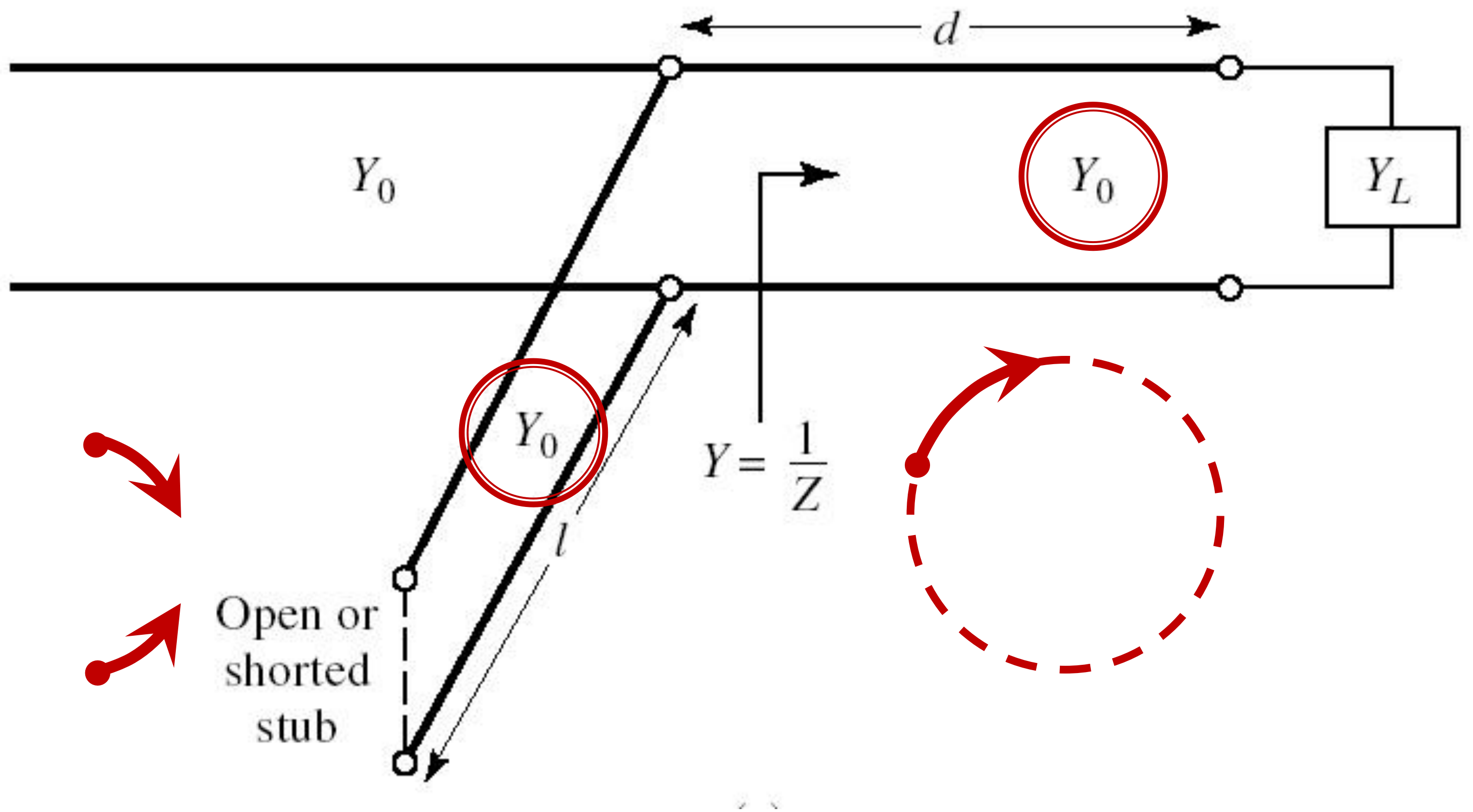
Impedance Matching with Stubs

Smith chart, r=1 and g=1



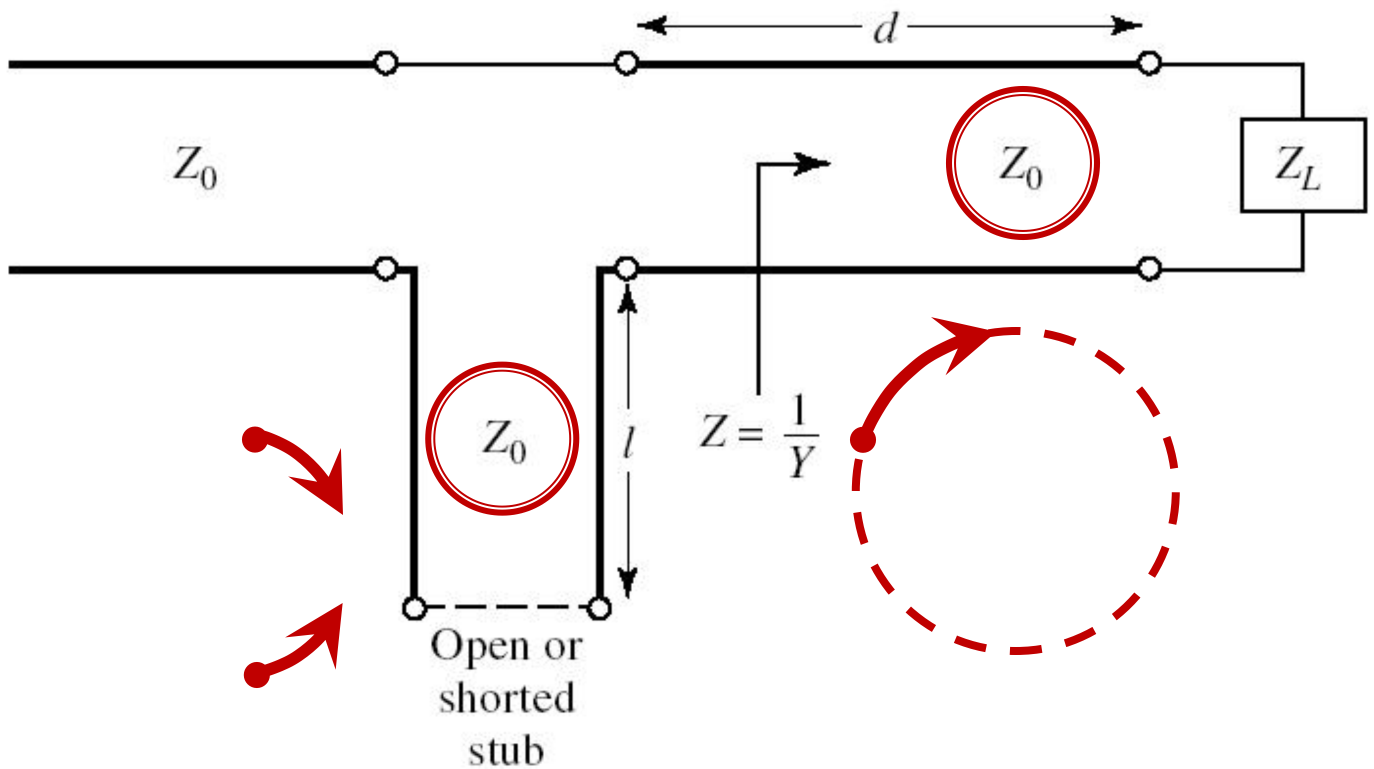
Single stub tuning

- Shunt Stub



Single stub tuning

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)

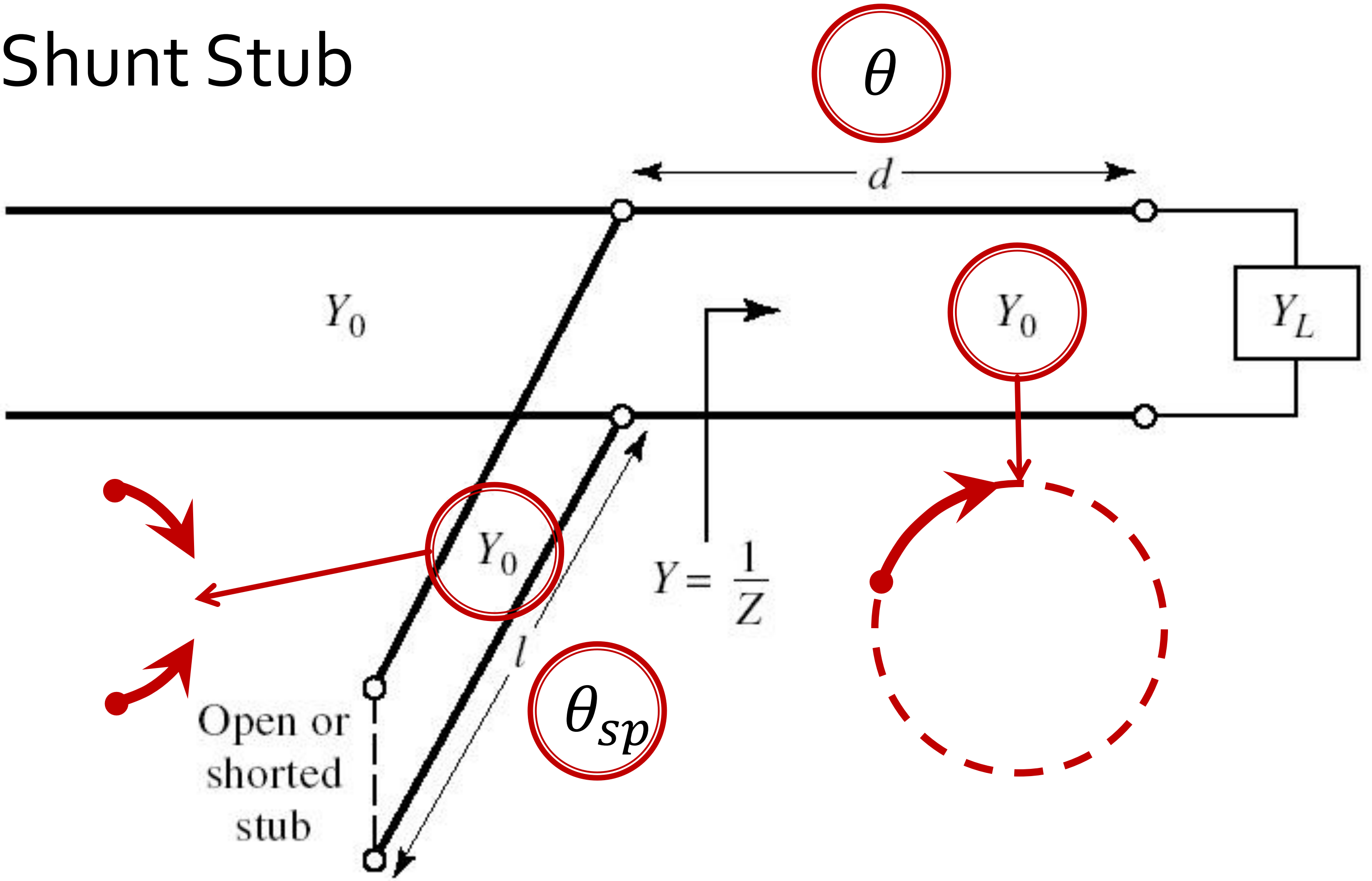


Analytical solutions

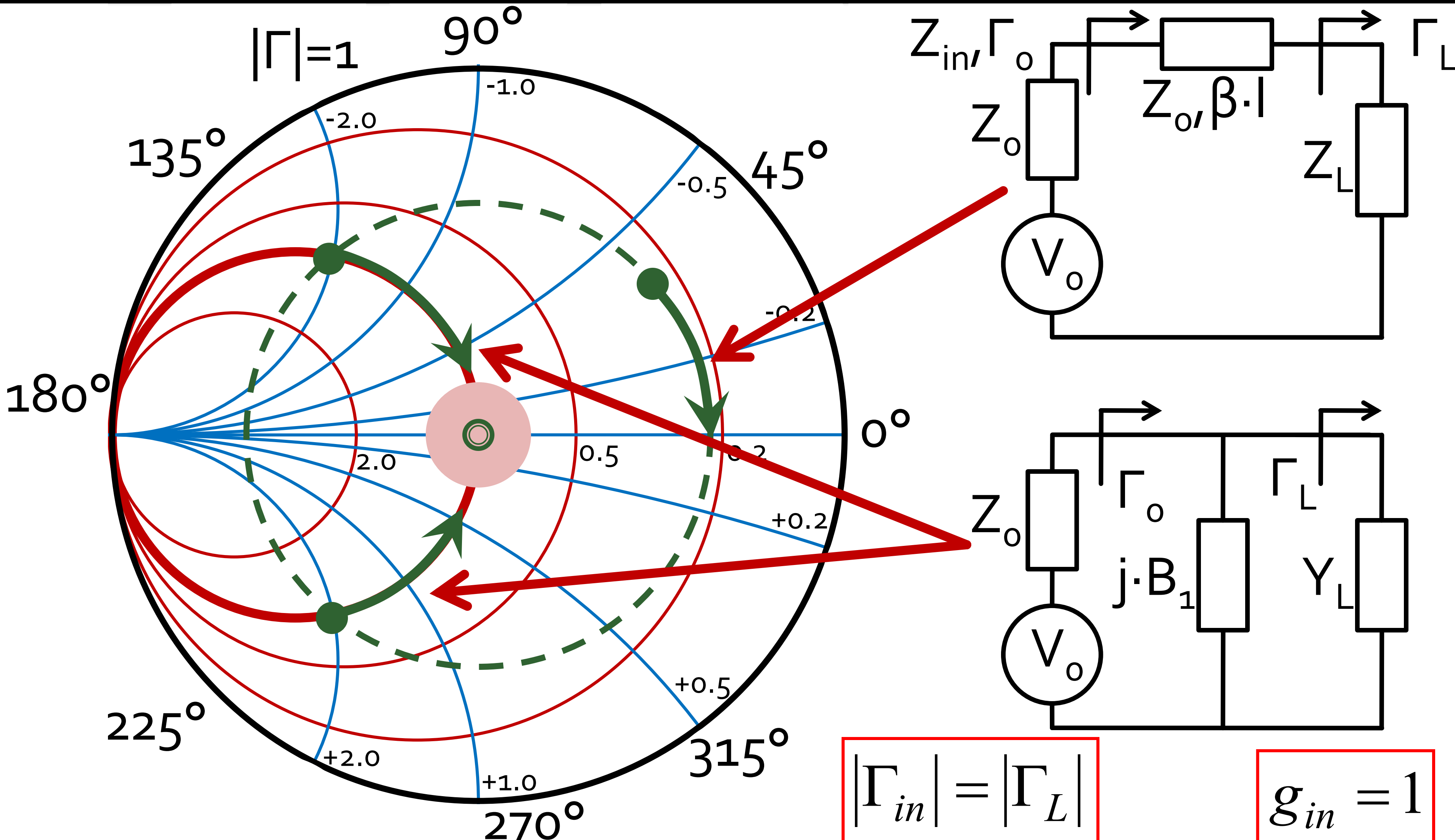
Exam / Project

Case 1, Shunt Stub

- Shunt Stub



Matching, series line + shunt susceptance



$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

$$g_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.593\angle 46.85°$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85° \qquad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

    $$(46.85° + 2\theta) = +126.35° \qquad \theta = +39.7° \qquad \operatorname{Im} y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

    $$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = -55.8°(+180°) \rightarrow \theta_{sp} = 124.2°$$

  - **"-" solution**

    $$(46.85° + 2\theta) = -126.35° \qquad \theta = -86.6°(+180°) \rightarrow \theta = 93.4°$$

    $$\operatorname{Im} y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = 55.8°$$

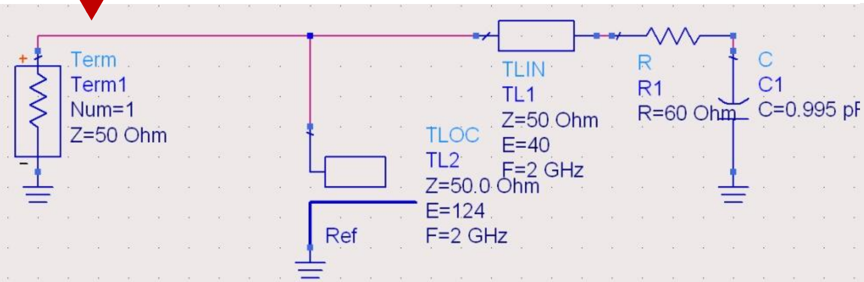
Analytical solution, usage

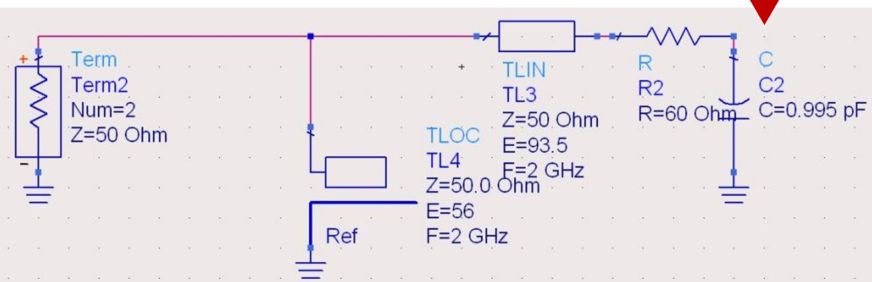
$$(\varphi + 2\theta) = \begin{cases} +126.35^\circ \\ -126.35^\circ \end{cases} \quad \theta = \begin{cases} 39.7^\circ \\ 93.4^\circ \end{cases} \quad \mathrm{Im}[y_S(\theta)] = \begin{cases} -1.472 \\ +1.472 \end{cases} \quad \theta_{sp} = \begin{cases} -55.8^\circ + 180^\circ = 124.2^\circ \\ +55.8^\circ \end{cases}$$

- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

$$l_1 = \frac{39.7^\circ}{360^\circ} \cdot \lambda = 0.110 \cdot \lambda \qquad l_1 = \frac{93.4^\circ}{360^\circ} \cdot \lambda = 0.259 \cdot \lambda$$

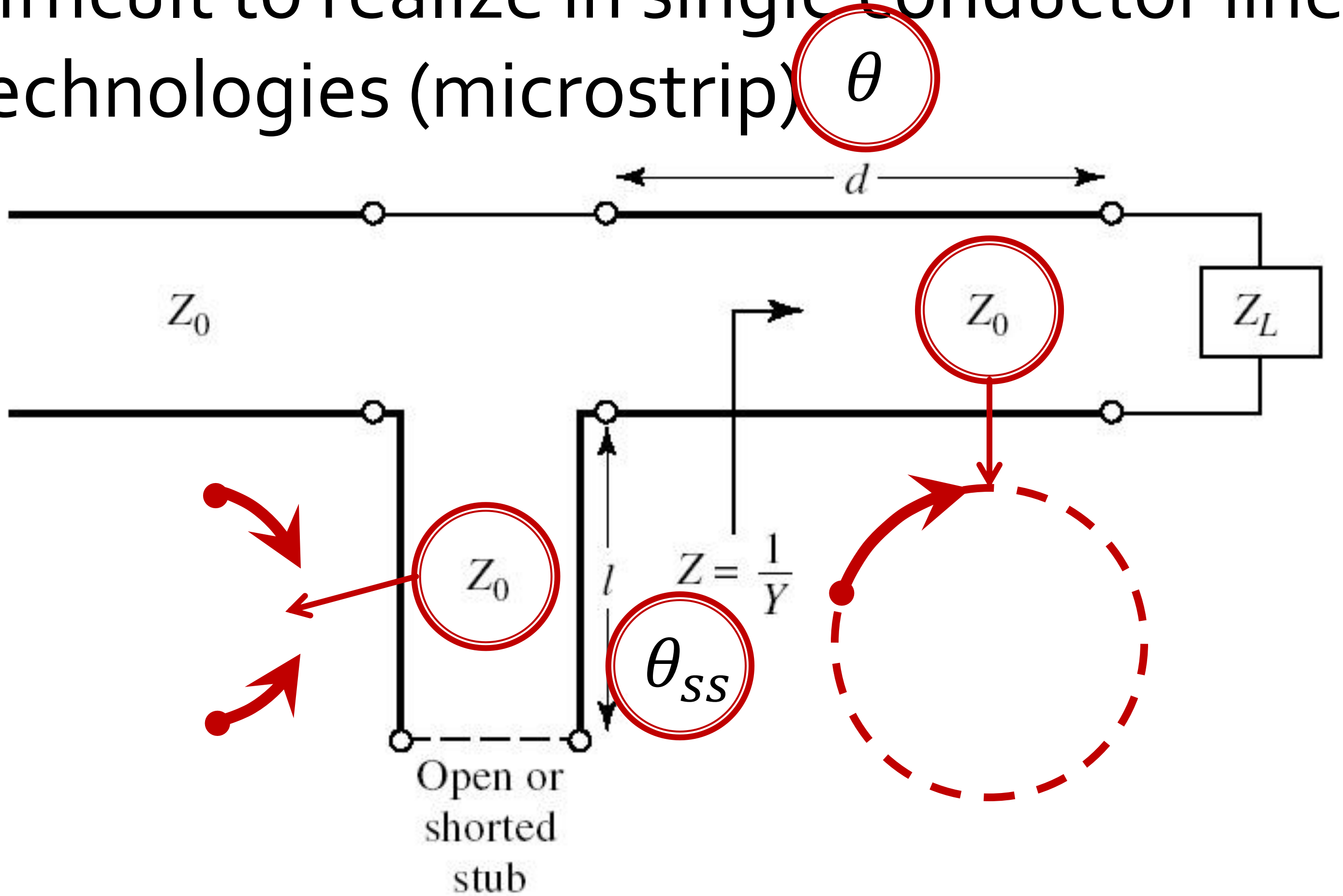
$$l_2 = \frac{124.2^\circ}{360^\circ} \cdot \lambda = 0.345 \cdot \lambda \qquad l_2 = \frac{55.8^\circ}{360^\circ} \cdot \lambda = 0.155 \cdot \lambda$$



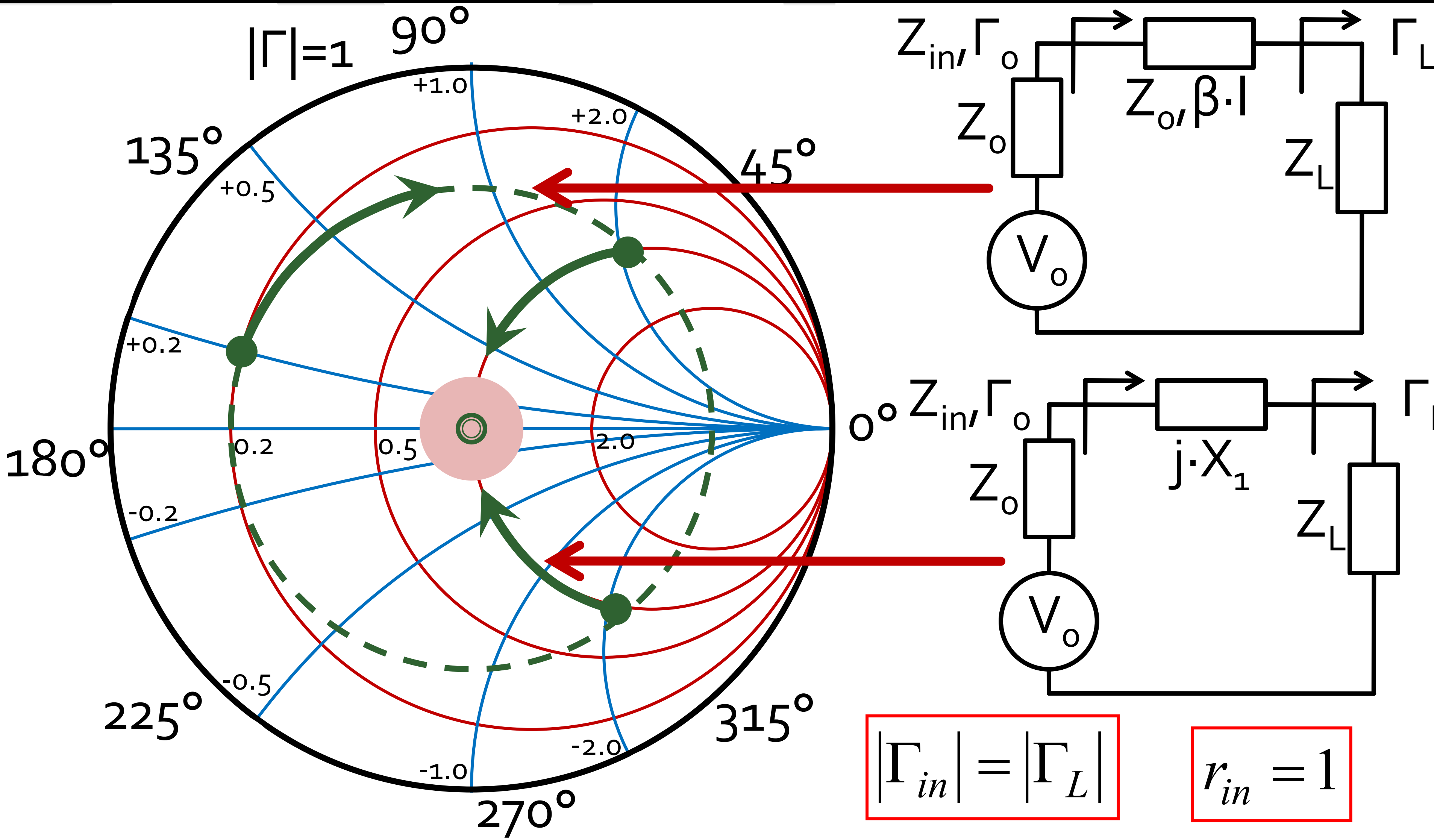


Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



Matching, series line + series reactance



$$|\Gamma_{in}| = |\Gamma_L|$$

$$r_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = |\Gamma_S|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.555\angle -29.92°$$

$$|\Gamma_S| = 0.555; \quad \varphi = -29.92° \qquad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation
  - **"+" solution**

    $$(-29.92° + 2\theta) = +56.28° \qquad \theta = 43.1° \qquad \operatorname{Im} z_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.335$$

    $$\theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = -36.8°(+180°) \rightarrow \theta_{ss} = 143.2°$$
  - **"-" solution**

    $$(-29.92° + 2\theta) = -56.28° \qquad \theta = -13.2°(+180°) \rightarrow \theta = 166.8°$$

    $$\operatorname{Im} z_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.335 \qquad \theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = 36.8°$$

Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +56.28° \\ -56.28° \end{cases} \quad \theta = \begin{cases} 43.1° \\ 166.8° \end{cases} \quad \mathrm{Im}[z_S(\theta)] = \begin{cases} +1.335 \\ -1.335 \end{cases} \quad \theta_{ss} = \begin{cases} -36.8° + 180° = 143.2° \\ +36.8° \end{cases}$$

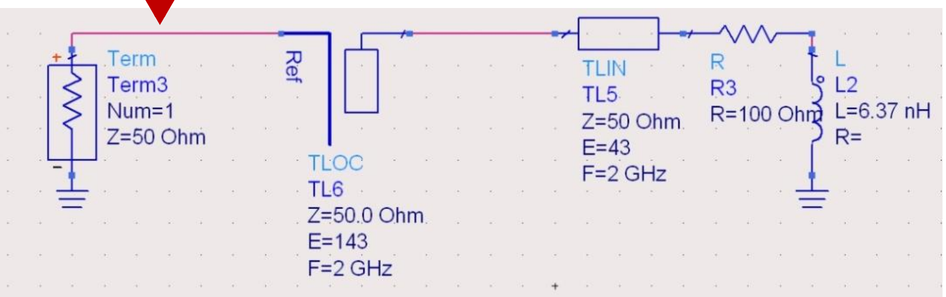
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

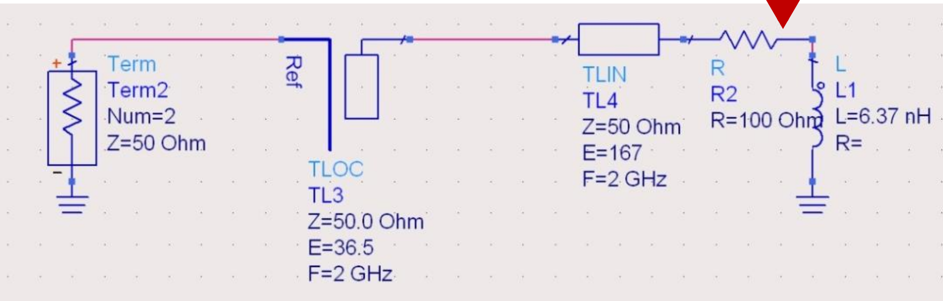
$$l_1 = \frac{43.1°}{360°} \cdot \lambda = 0.120 \cdot \lambda$$

$$l_2 = \frac{143.2°}{360°} \cdot \lambda = 0.398 \cdot \lambda$$

$$l_1 = \frac{166.8°}{360°} \cdot \lambda = 0.463 \cdot \lambda$$

$$l_2 = \frac{36.8°}{360°} \cdot \lambda = 0.102 \cdot \lambda$$





Stub, observations

- adding or subtracting **180°** (λ/2) doesn't change the result (full rotation around the Smith Chart)

$$E = \beta \cdot l = \pi = 180^\circ \qquad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbf{N}$$

  - if the lines/stubs result with **negative** "length"/ "electrical length" we add λ/2 / 180° to obtain physically realizable lines
- adding or subtracting **90°** (λ/4) change the stub impedance:

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \quad \Leftrightarrow \quad Z_{in,g} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

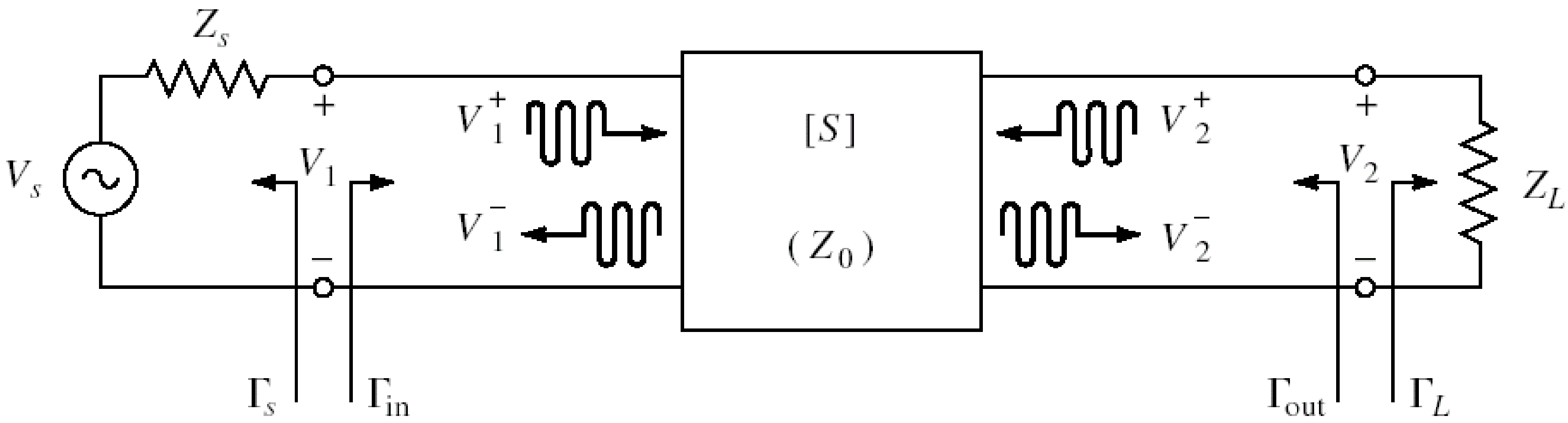
  - for the stub we can add or subtract 90° (λ/4) while in the same time changing **open-circuit** ⇔ **short-circuit**

Microwave Amplifiers

Course Topics

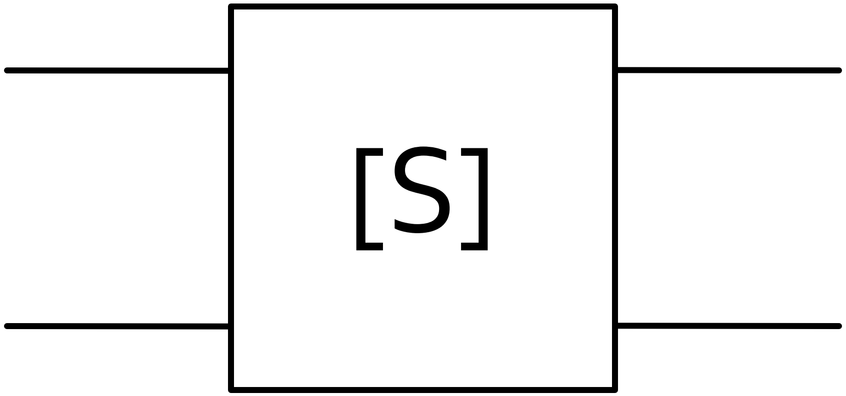
- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

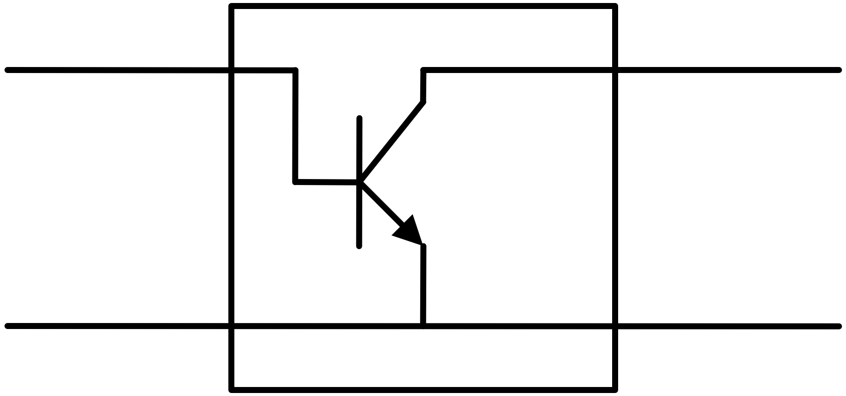
Amplifier as two-port

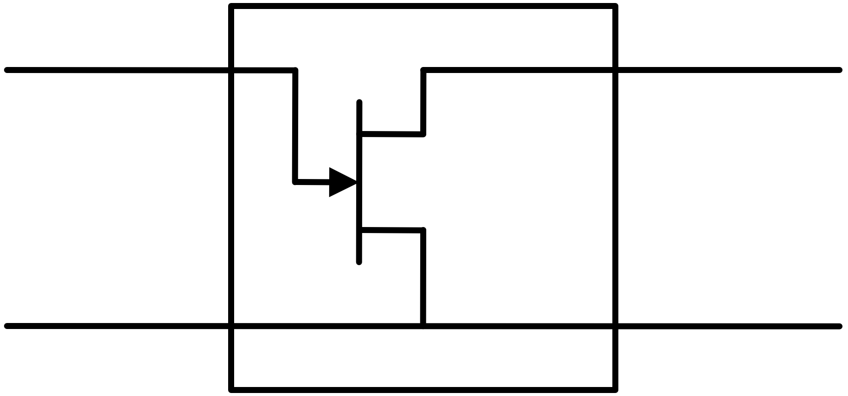


- Charaterized with S parameters
- normalized at Zo (implicit 50Ω)
- Datasheets: S parameters for specific bias conditions

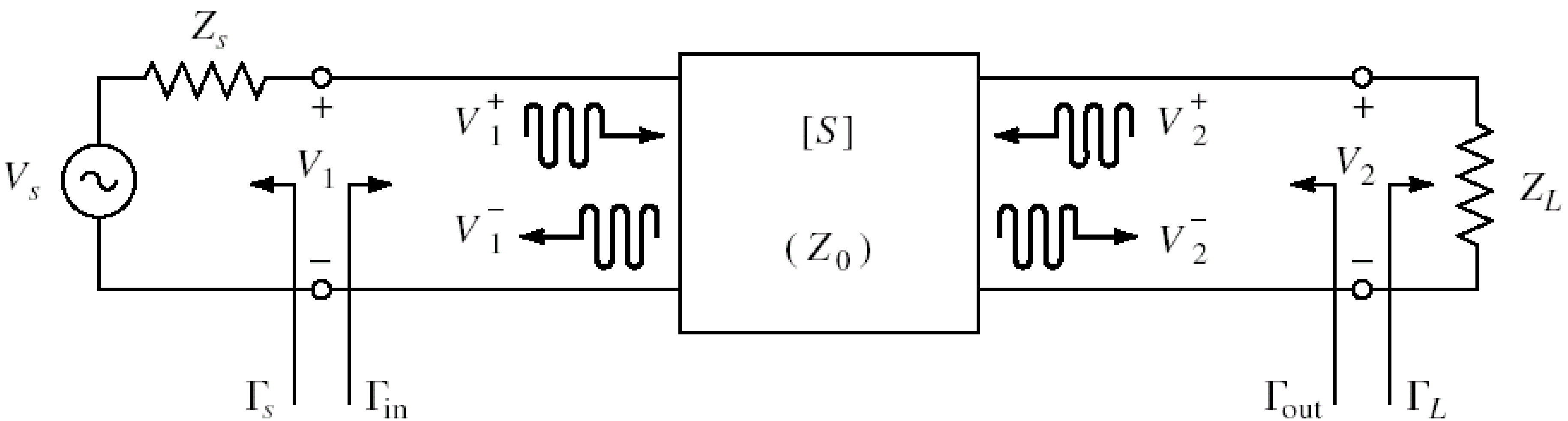
S parameters for transistors







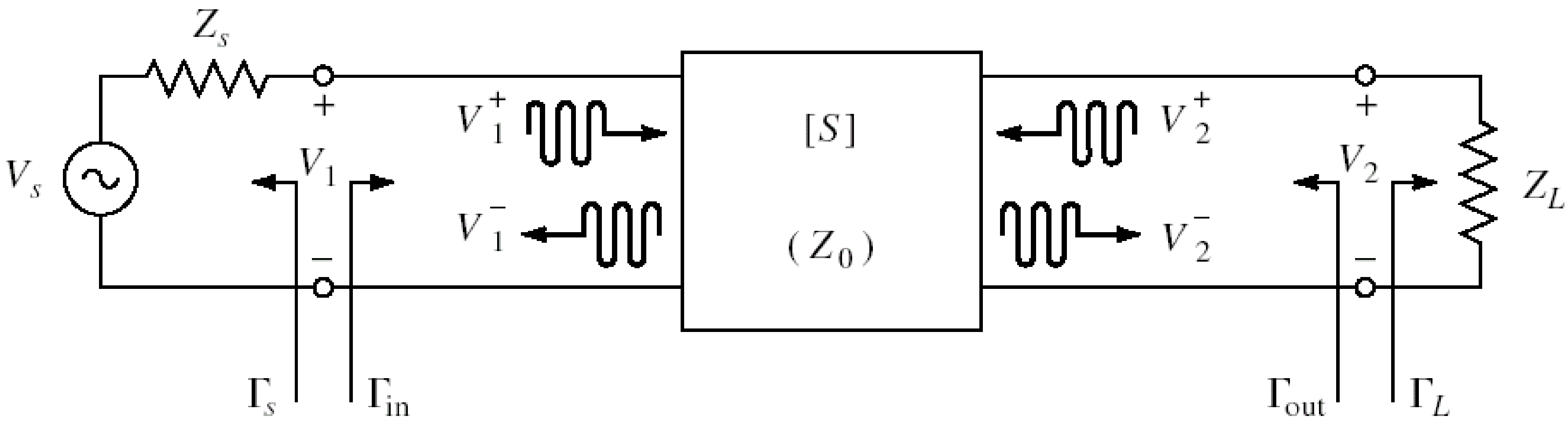
Amplifier as two-port



$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

Amplifier as two-port

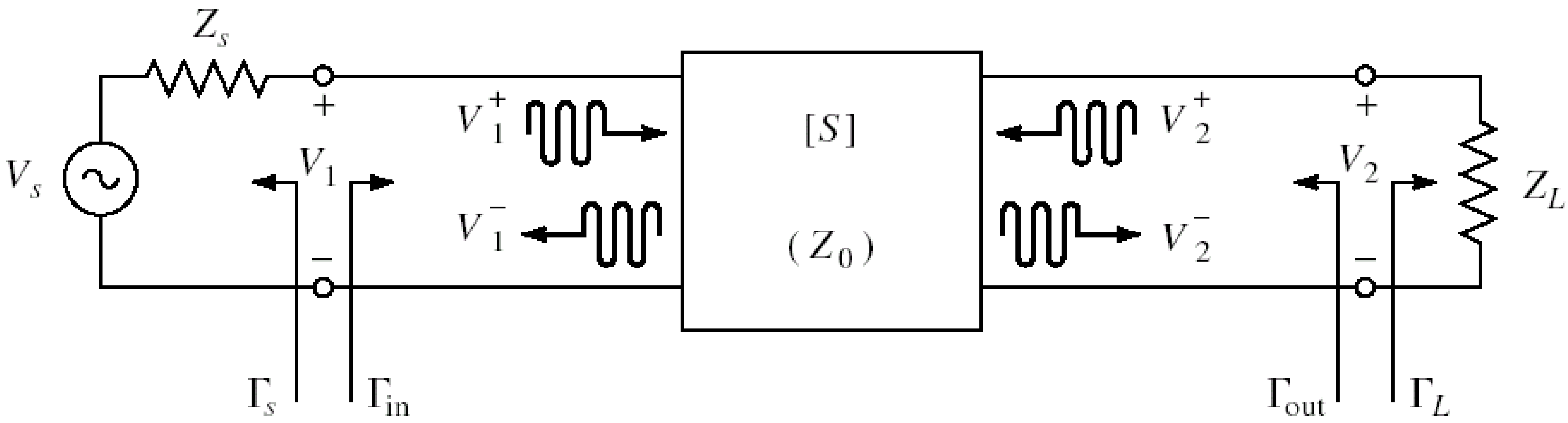


- For an amplifier two-port we are interested in:
  - stability
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Microwave Amplifiers

Stability

Amplifier as two-port



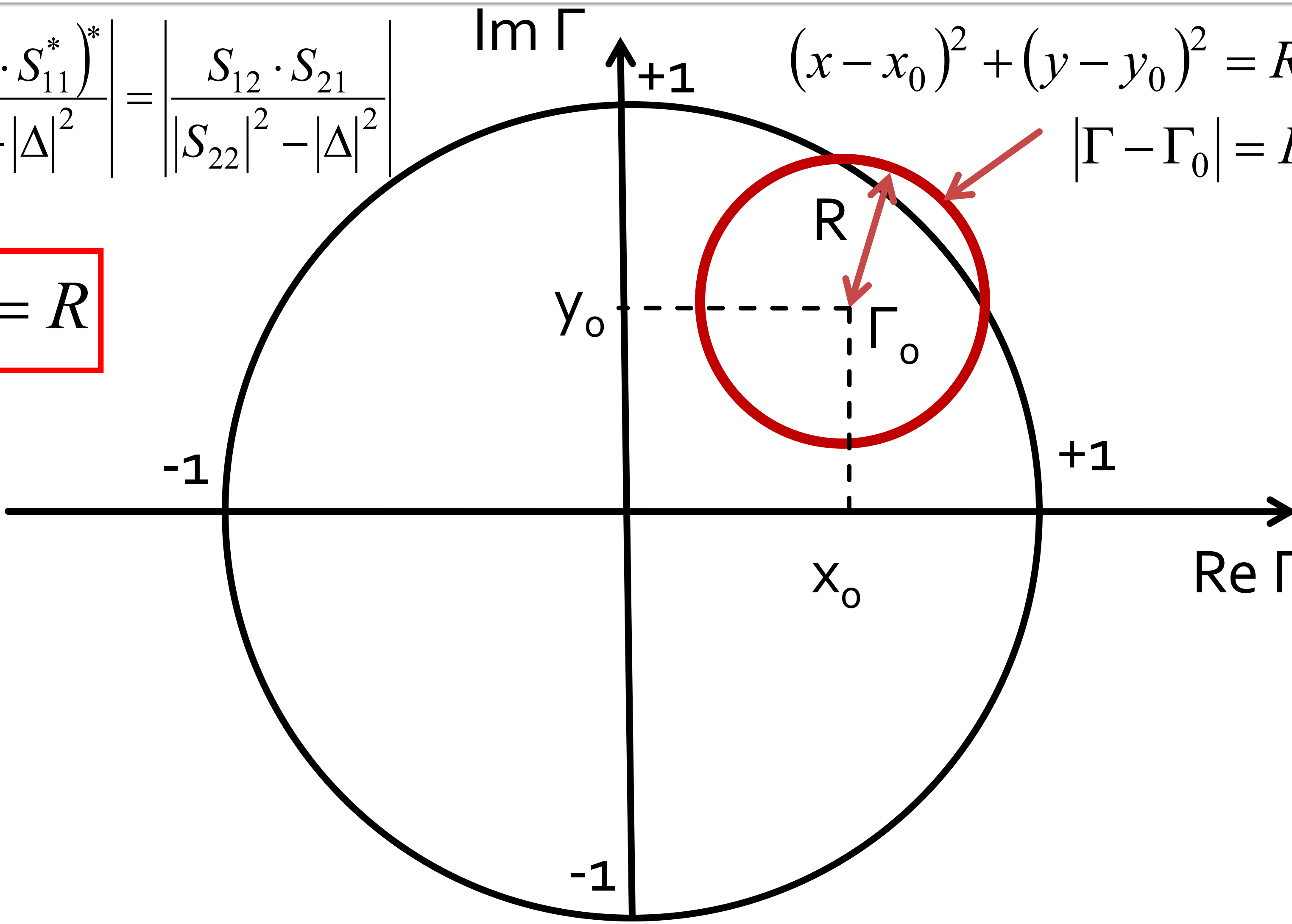
- For an amplifier two-port we are interested in:
  - **stability**
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Stability

$$\left| \Gamma_L - \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2} \right| = \left| \frac{S_{12} \cdot S_{21}}{\left|S_{22}\right|^2 - \left|\Delta\right|^2} \right|$$

$$\left(x - x_0\right)^2 + \left(y - y_0\right)^2 = R^2$$

$$\left|\Gamma - \Gamma_0\right| = R$$



$$\left|\Gamma - \Gamma_0\right| = R$$

Output stability circle (CSOUT)

$$\left|\Gamma_L - \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}\right| = \left|\frac{S_{12} \cdot S_{21}}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}\right| \qquad \left|\Gamma_L - C_L\right| = R_L$$

- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_L for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- This circle is the **output stability circle** (Γ_L)

$$C_L = \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2} \qquad R_L = \frac{\left|S_{12} \cdot S_{21}\right|}{\left|\left|S_{22}\right|^2 - \left|\Delta\right|^2\right|}$$

Input stability circle (CSIN)

- Similarly

$$|\Gamma_{out}| = 1 \qquad \left| S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S} \right| = 1$$

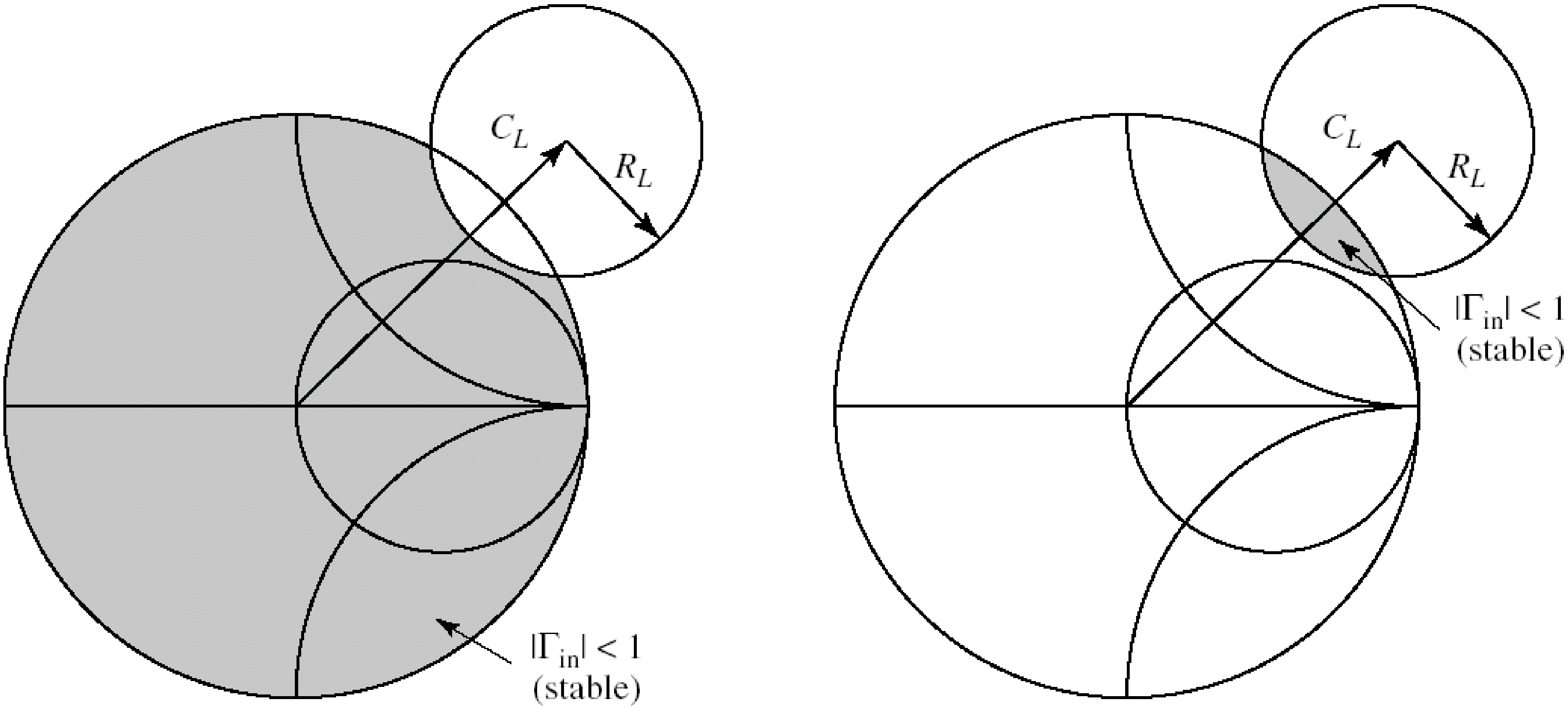
- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_S for the **limit between stability and instability** ($|\Gamma_{out}| = 1$)
- This circle is the **input stability circle** (Γ_S)

$$C_S = \frac{\left(S_{11} - \Delta \cdot S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2} \qquad R_S = \frac{|S_{12} \cdot S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|}$$

Output stability circle (CSOUT)

- The **output stability circle** represents the locus of Γ_L for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- The circle divides the complex planes in two areas, the **inside** and the **outside** of the circle
- The two areas will represent the locus of Γ_L for stability ($|\Gamma_{in}|<1$) / instability ($|\Gamma_{in}|>1$)

Output stability circle (CSOUT)



(a) (b)

- Two cases possible: (a) stable outside/ (b) stable inside

Identification of the stability / instability regions

- Output stability circle
  - **|S11| < 1** → the center of the Smith chart on which Γ_L is represented is a **stable point**, so it's placed in the stability region (most often situation)
  - |S11| > 1 → the center of the Smith chart on which Γ_L is represented is an **unstable point**, so it's placed in the instability region
- Input stability circle
  - **|S22| < 1** → the center of the Smith chart on which Γ_S is represented is a **stable point**, so it's placed in the stability region (most often situation)
  - |S22| > 1 → the center of the Smith chart on which Γ_S is represented is a **unstable point**, so it's placed in the instability region

Solution + region identification

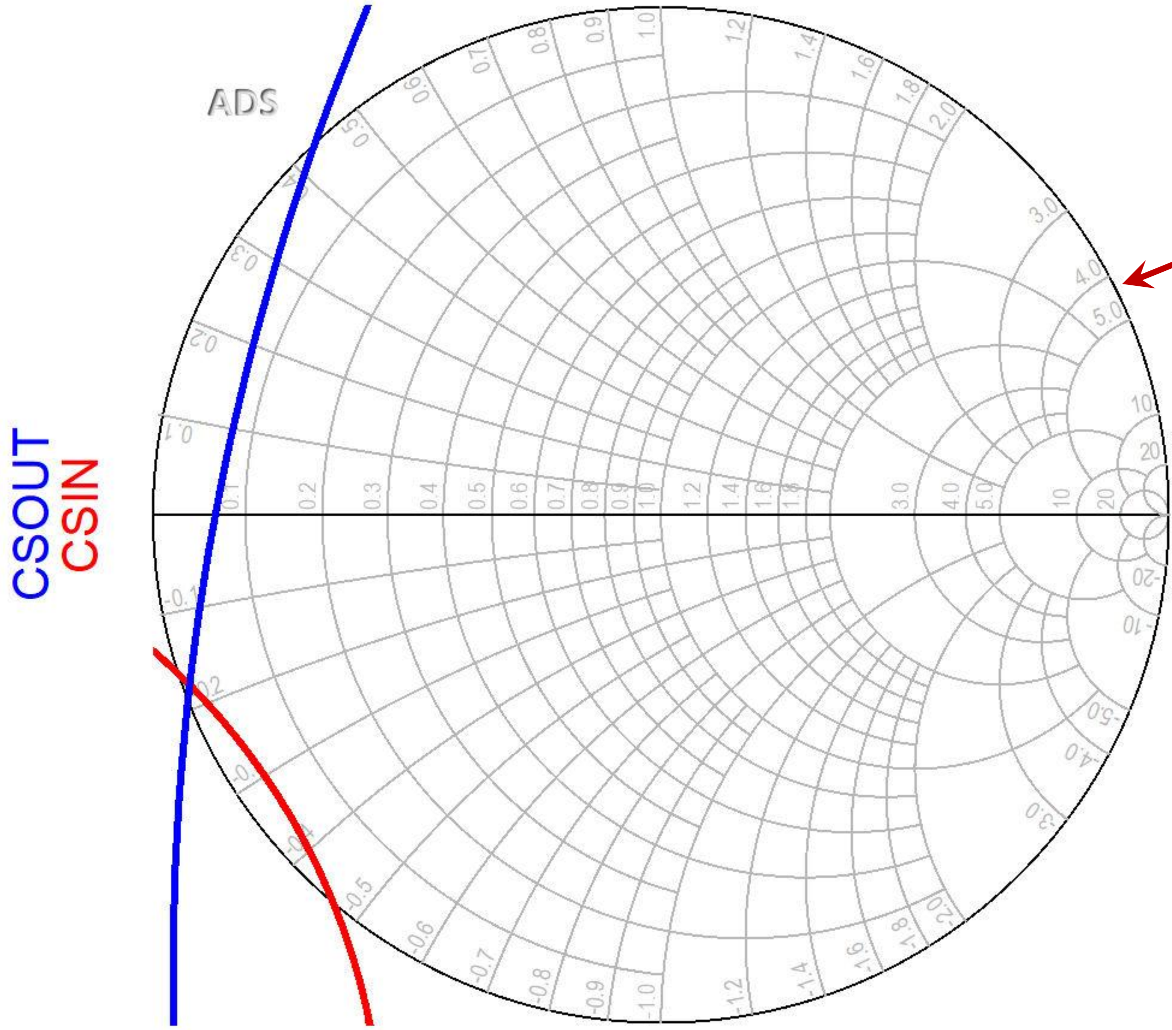
- S parameters
  - S11 = -0.483+0.42·j
  - S12 = 0.111-0.043·j
  - S21 = 3.042+0.872·j
  - S22 = -0.182+0.123·j
- | S11 | = 0.64 < 1
- $|C_L| < R_L$, 0∈CSOUT
- The center of the Smith chart is placed inside the output stability circle (0∈CSOUT) and is a stable point (| S11 | < 1)
  - the inside of the output stability circle – stability region
  - the outside of the output stability circle – instability region

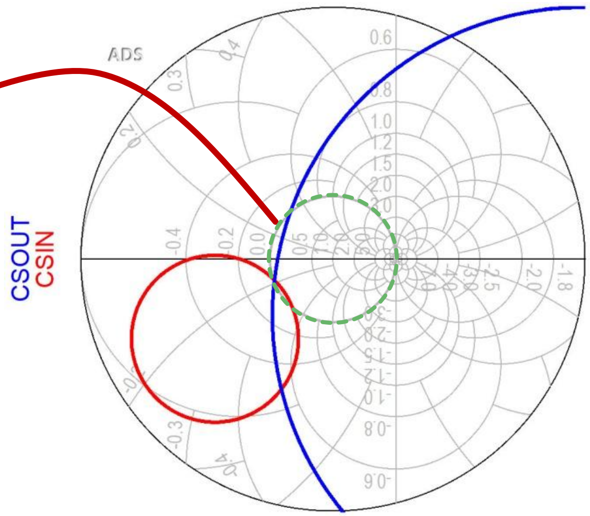
$$C_L = \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2} = 3.931 - 0.897 \cdot j$$

$$\left|C_L\right| = 4.032$$

$$R_L = \frac{\left|S_{12} \cdot S_{21}\right|}{\left|\left|S_{22}\right|^2 - \left|\Delta\right|^2\right|} = 4.891$$

ADS



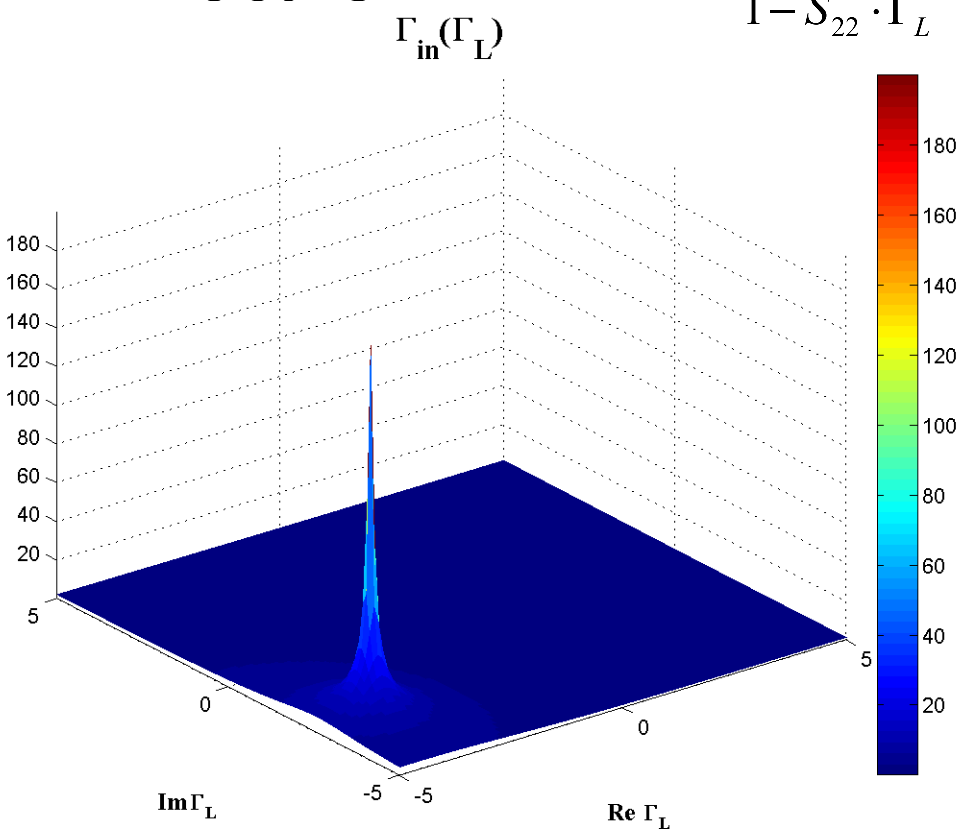


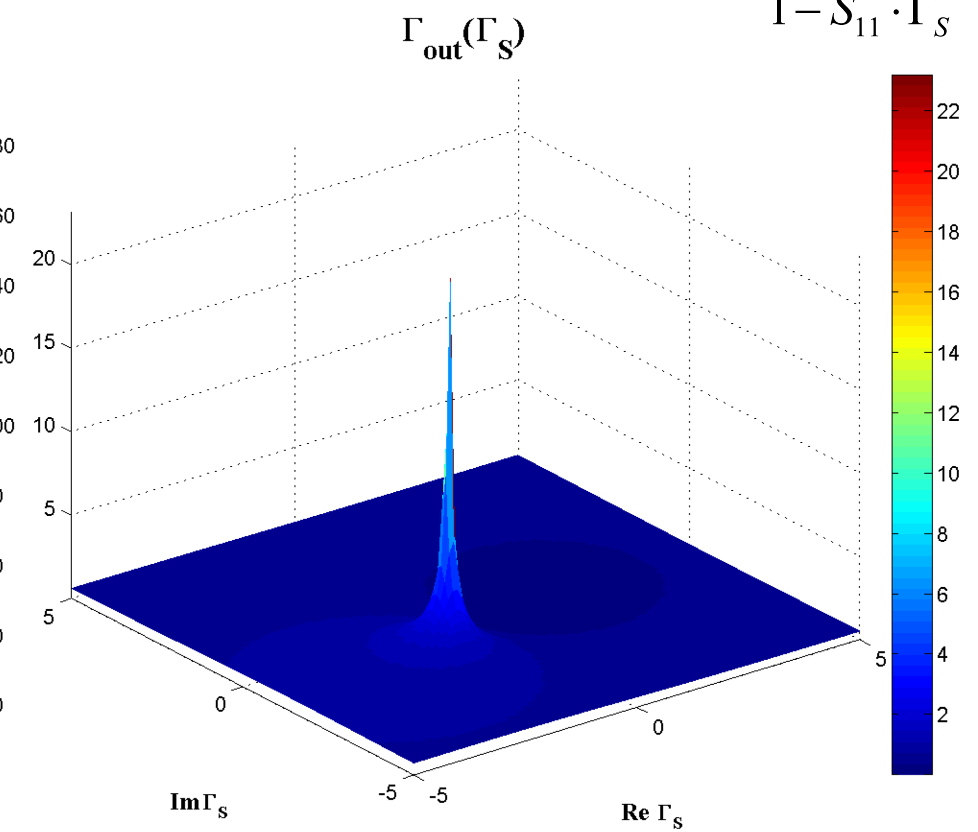
3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

- High variations -> we change to z logarithmic scale

$$\Gamma_{in}(\Gamma_L) = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

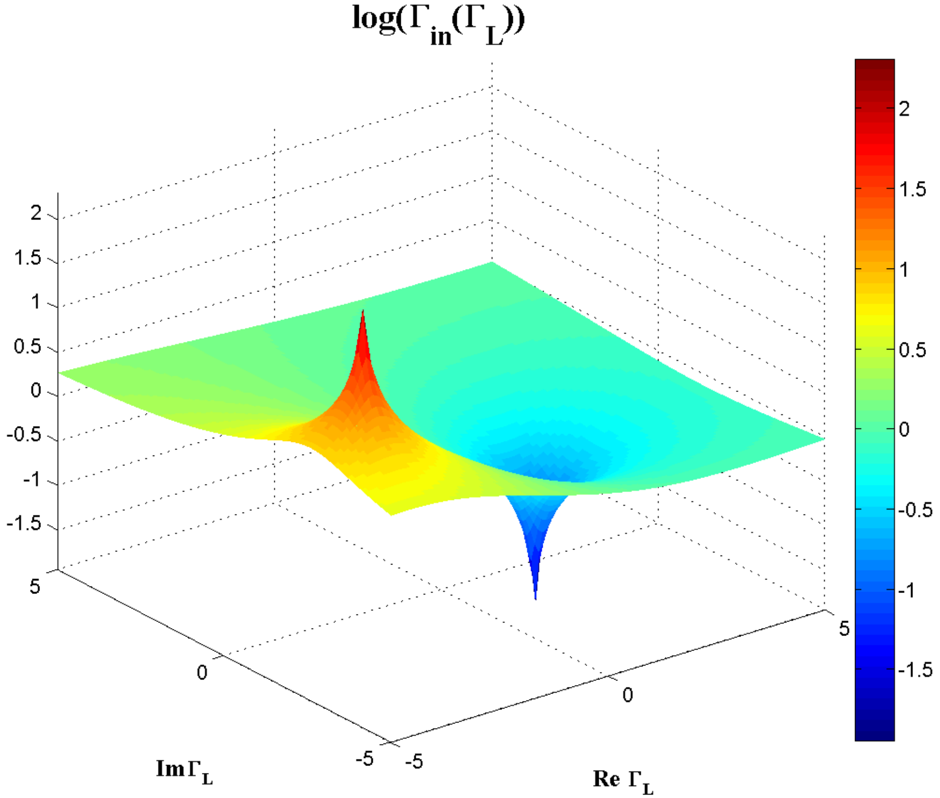
$$\Gamma_{out}(\Gamma_S) = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

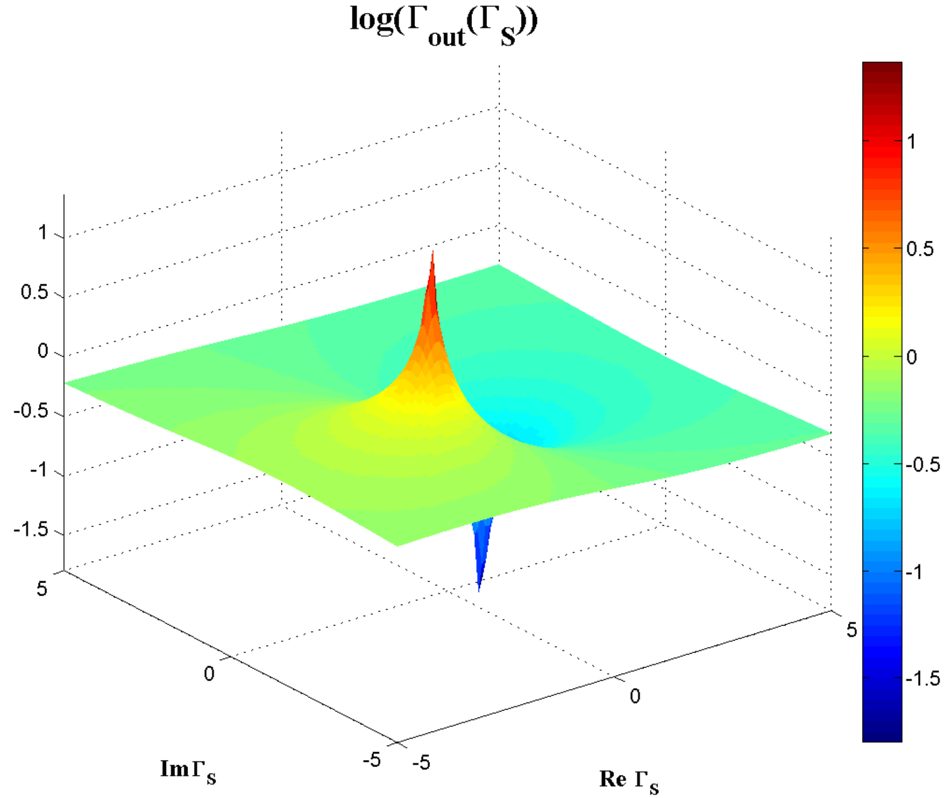




3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

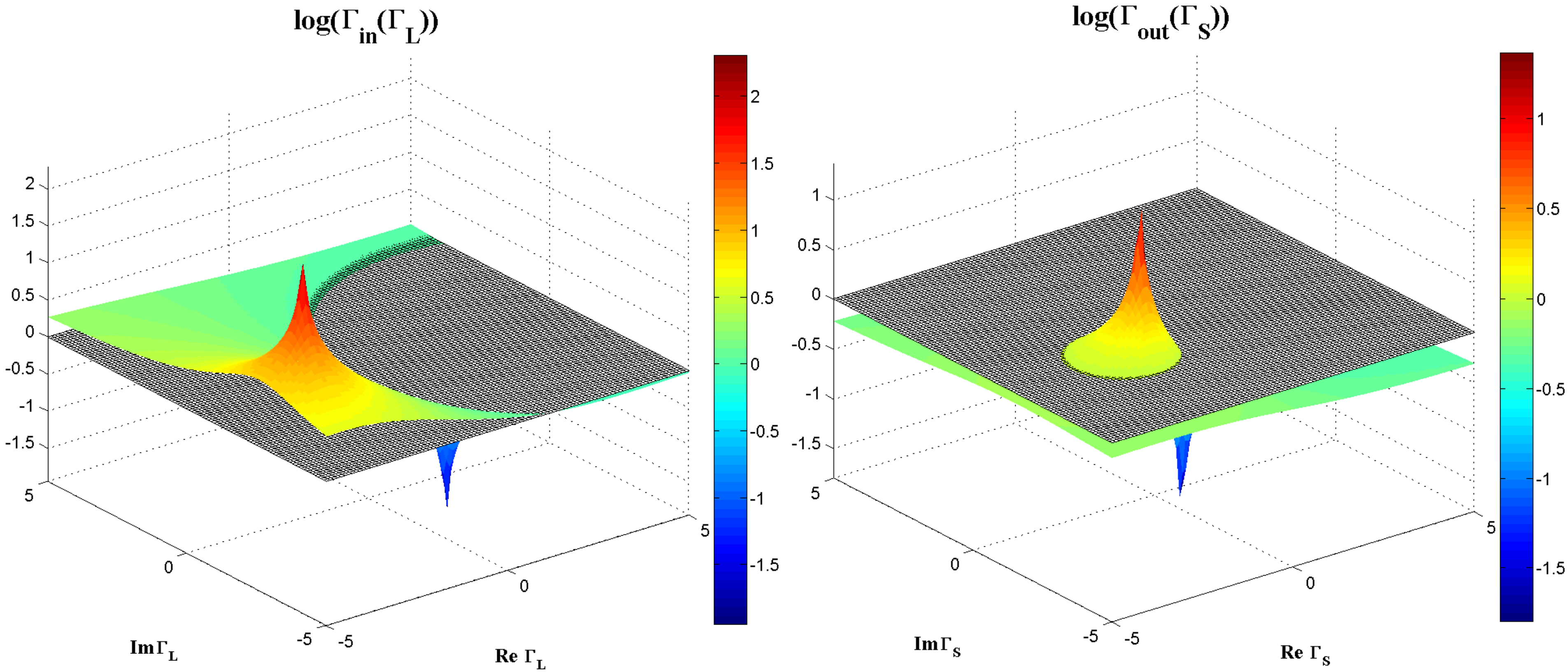
- $\log_{10}|\Gamma_{in}|$, $\log_{10}|\Gamma_{out}|$



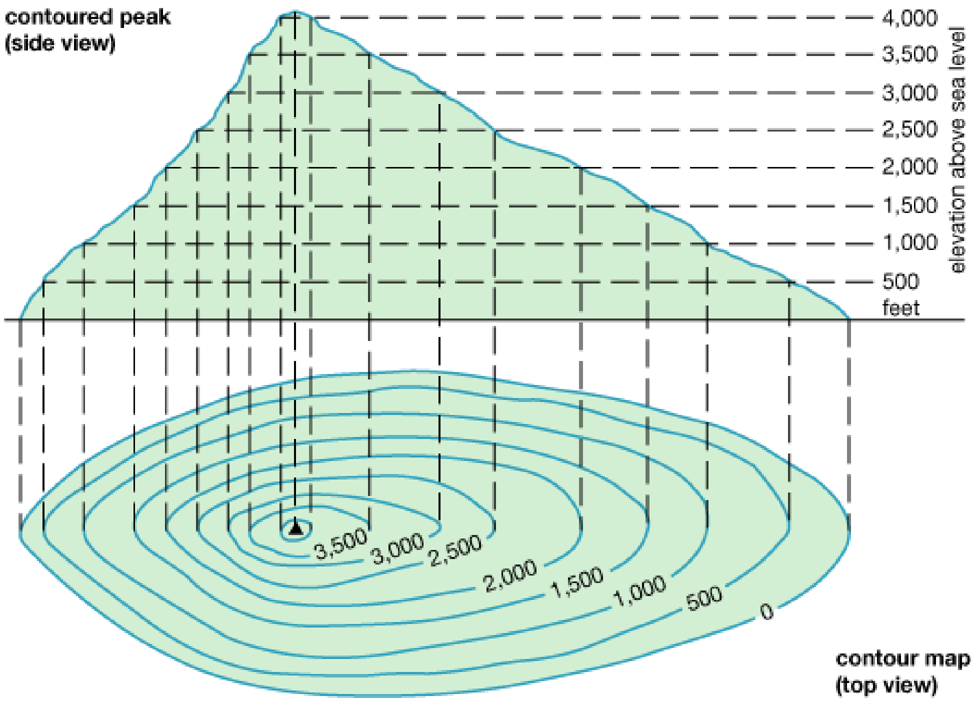


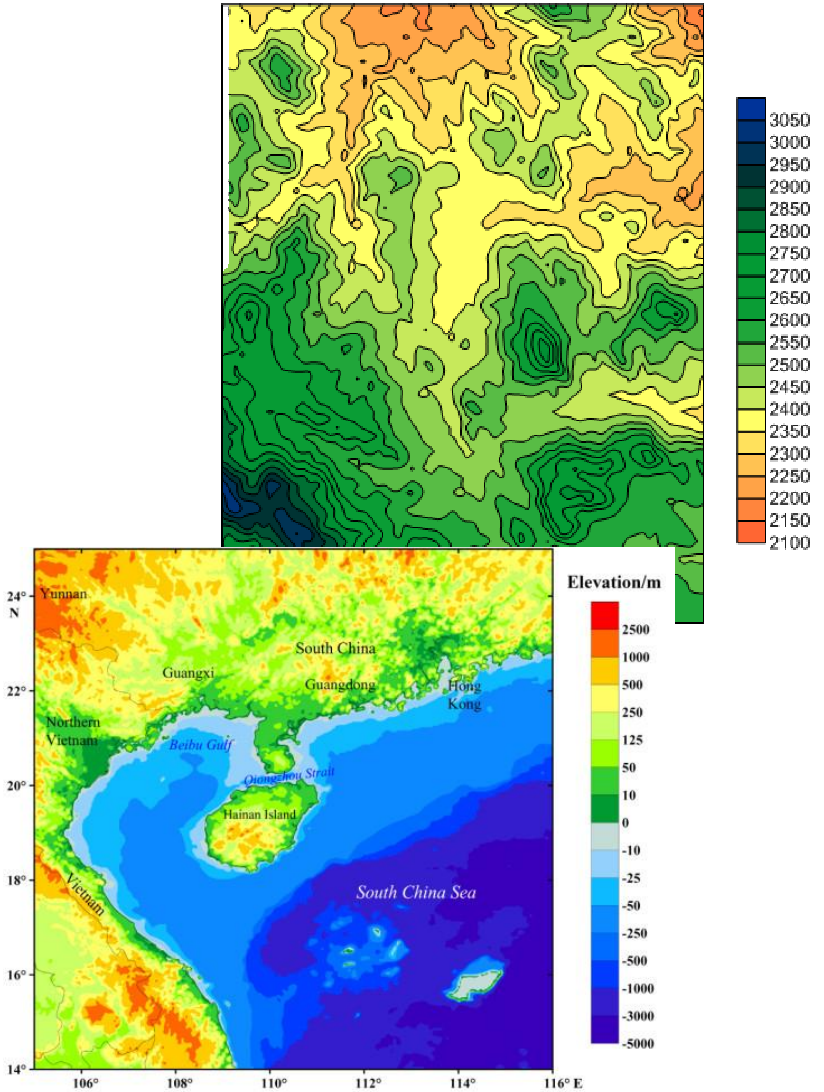
3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$, $|\Gamma|=1$

- $|\Gamma| = 1 \rightarrow \log_{10}|\Gamma| = 0$, the intersection with the plane z = 0 is a circle

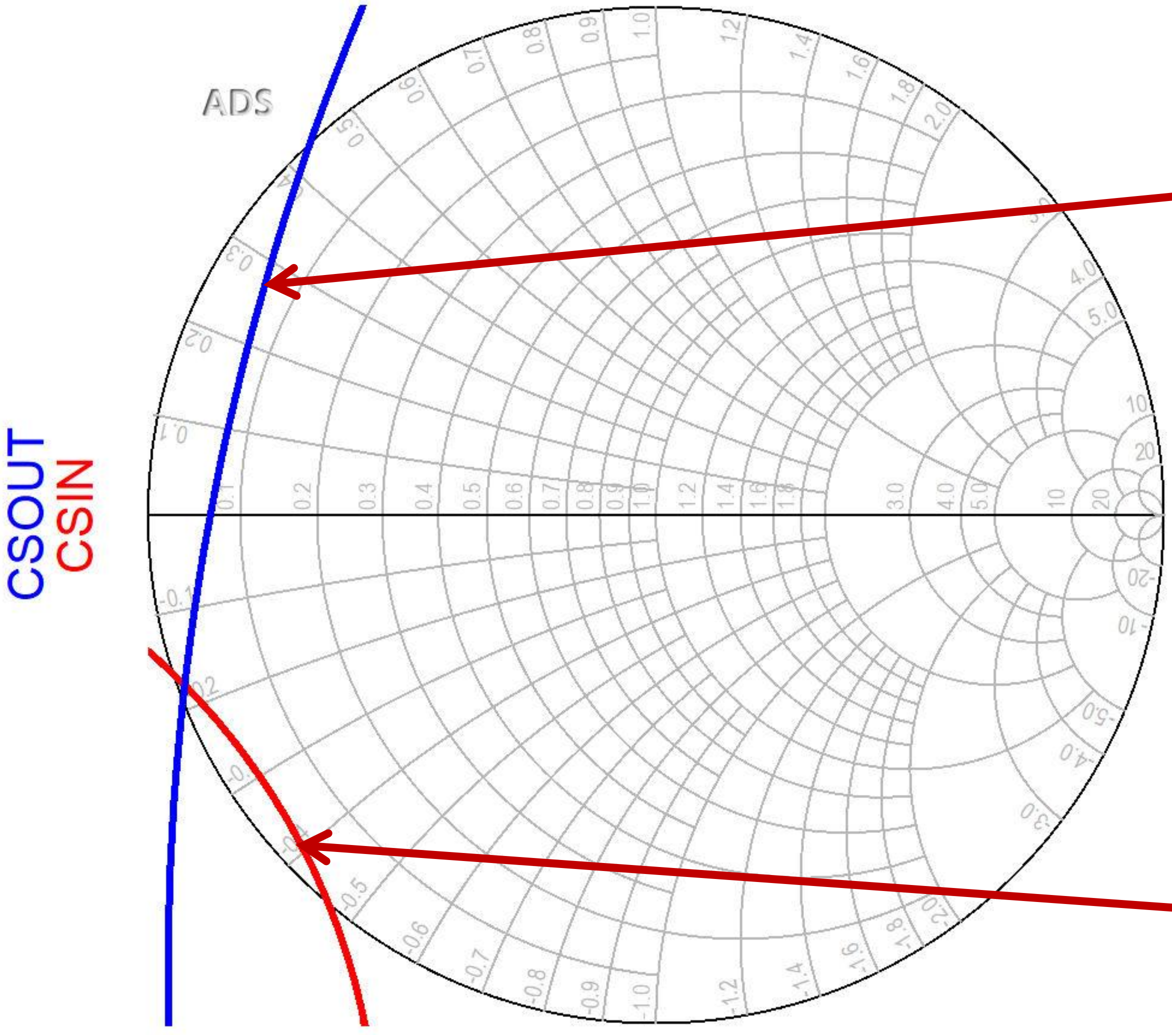


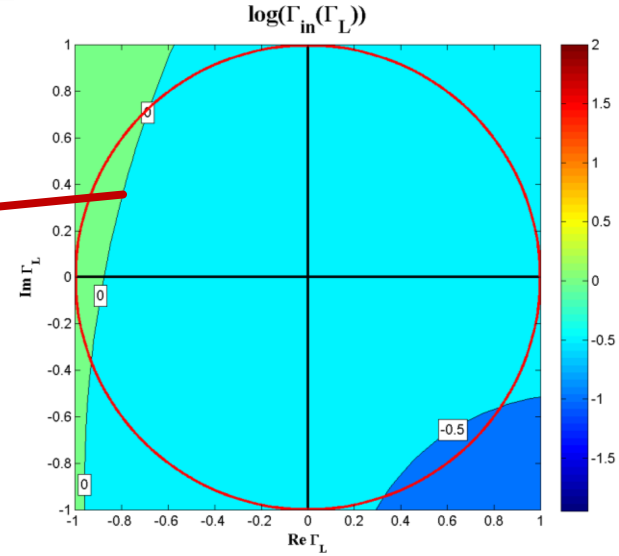
Contour map/lines

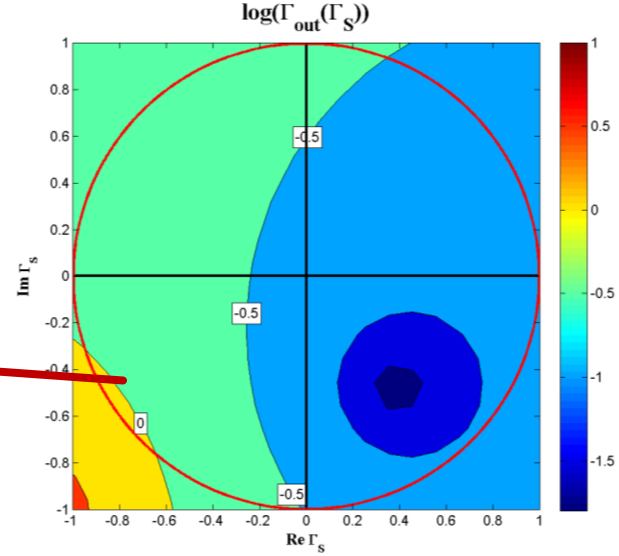




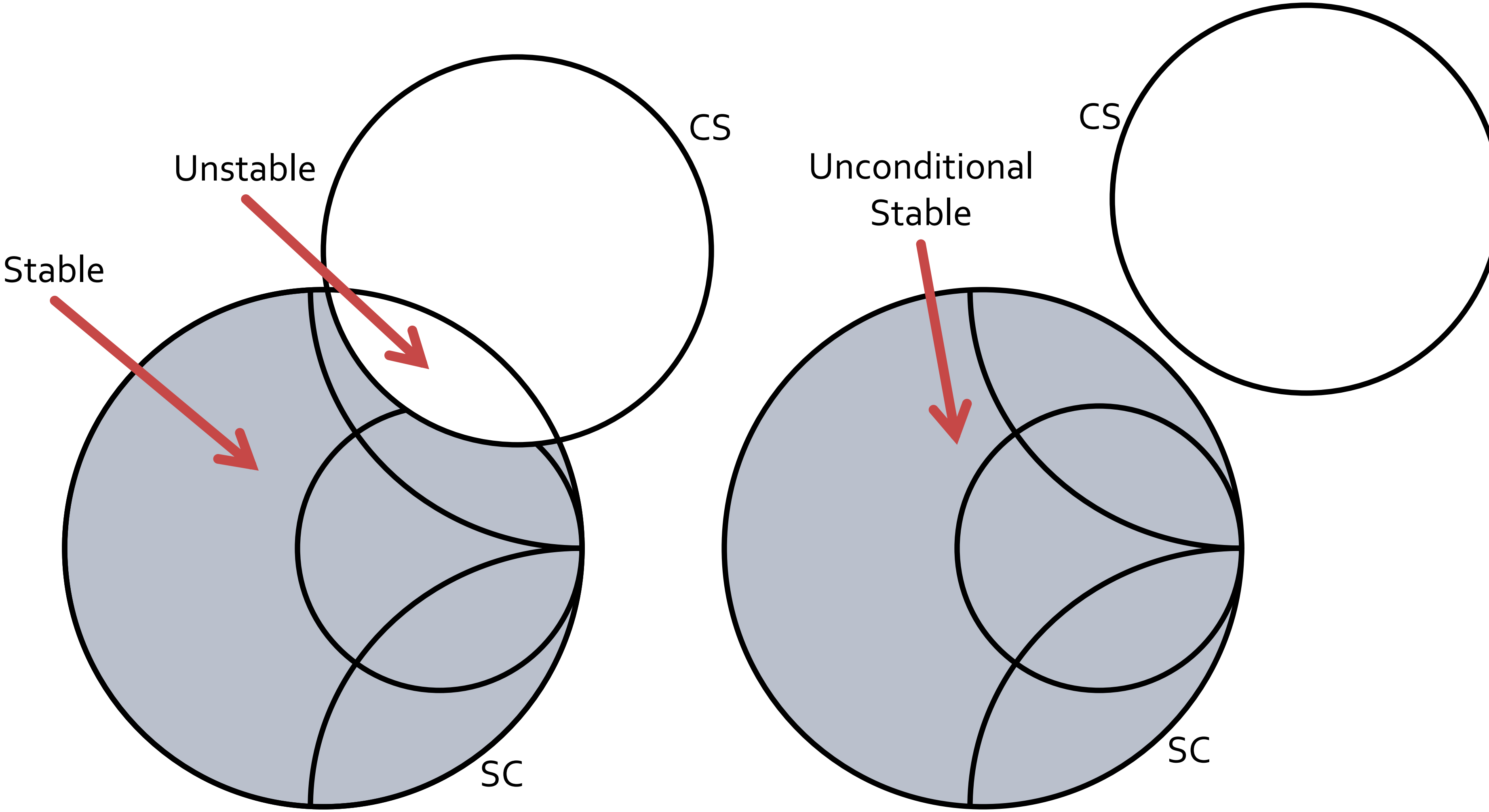
CSIN, CSOUT


ADS
CSOUT
CSIN

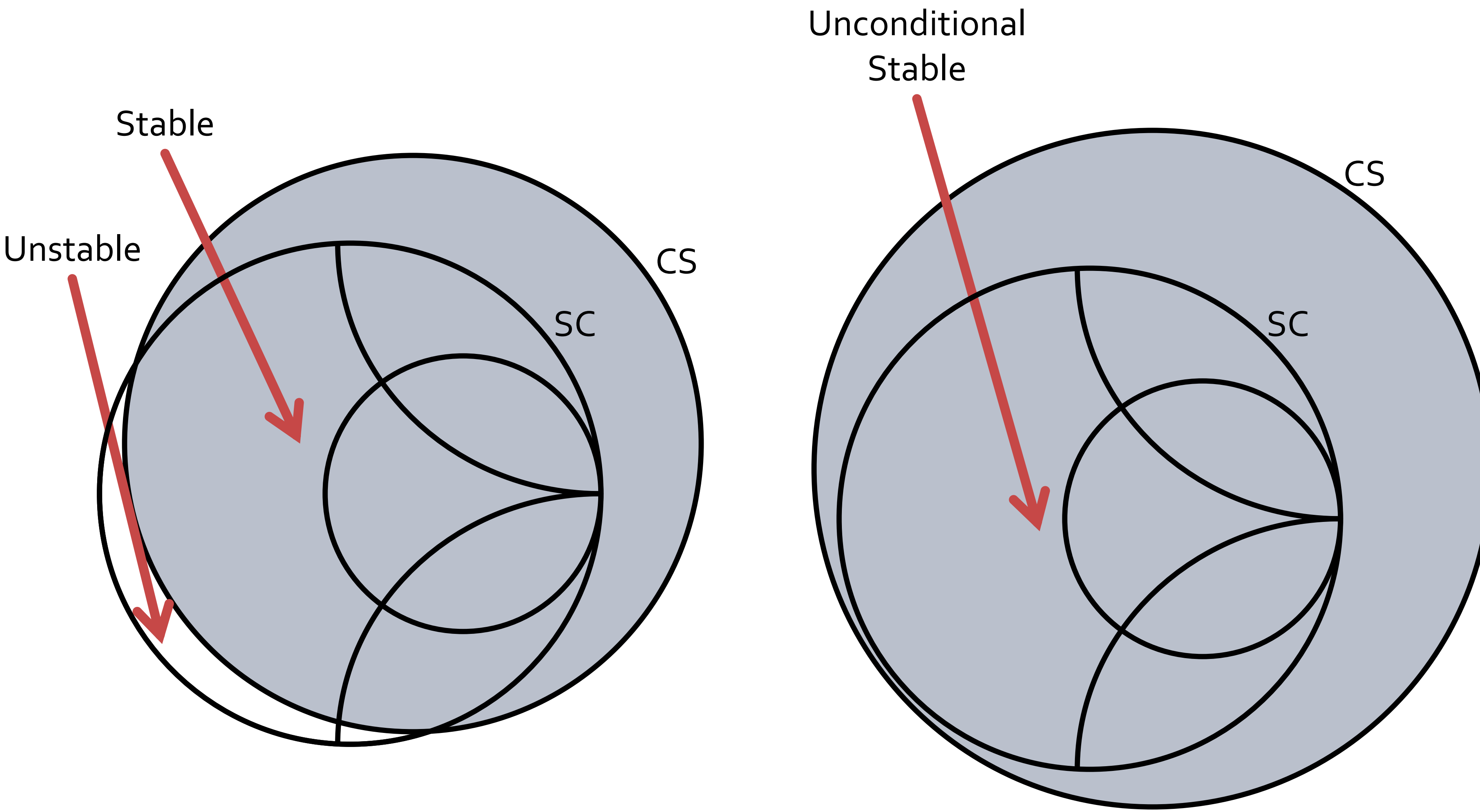

log(Γin(ΓL))
Im ΓL
Re ΓL
0
-0.5


log(Γout(ΓS))
Im ΓS
Re ΓS
0
-0.5

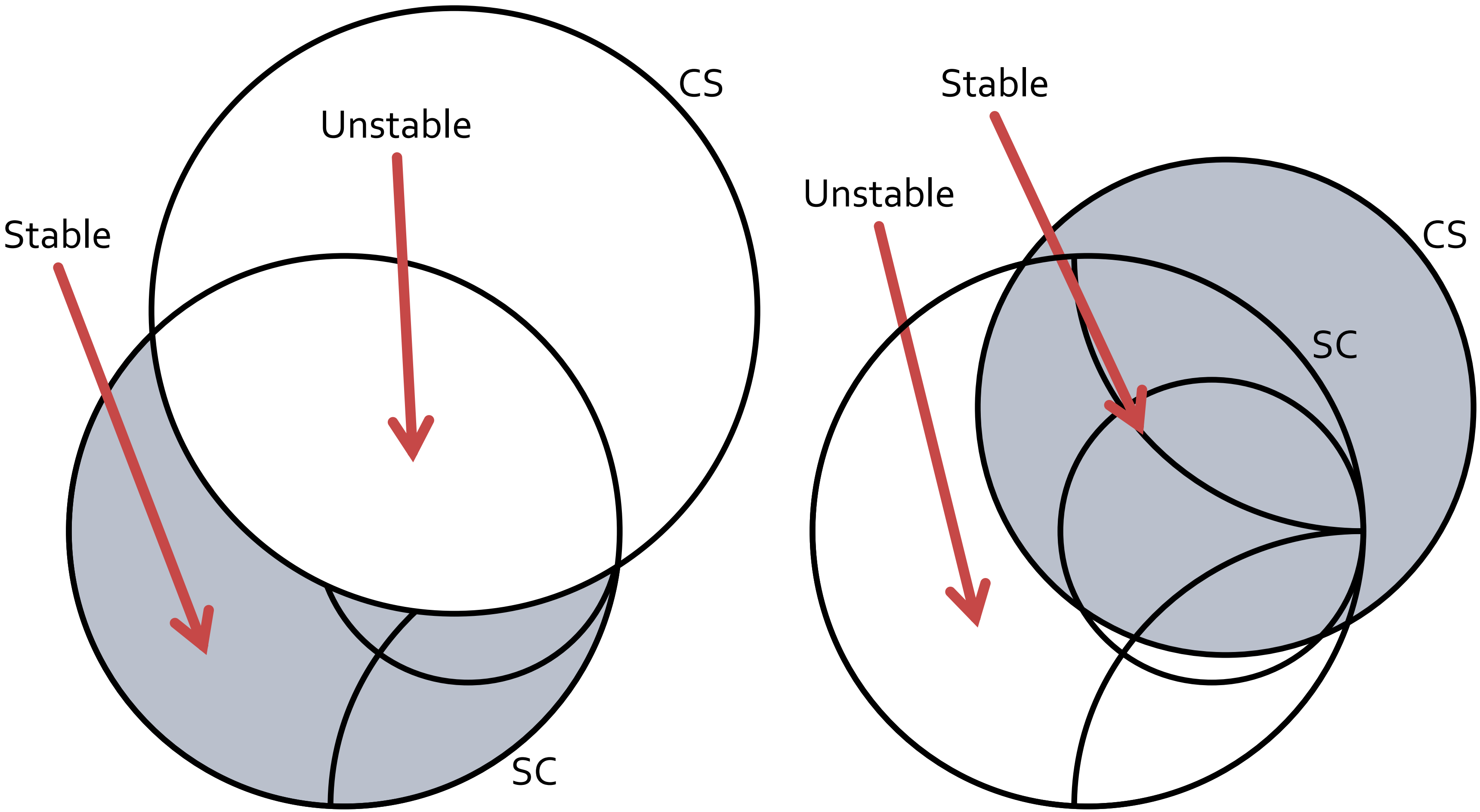
Several possible positioning



Several possible positioning



(Quite) Rare positioning



Stability

- **Unconditional stability:** the circuit is unconditionally stable if $|\Gamma_{in}|<1$ and $|\Gamma_{out}|<1$ for **any** passive impedance of the load/source
- **Conditional stability:** the circuit is conditionally stable if $|\Gamma_{in}|<1$ and $|\Gamma_{out}|<1$ only for **some** passive impedance of the load/source
  - passive impedance of the load/source <-> interior of the Smith Chart (radius 1 circle in the complex plane)

Rollet's condition

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|}$$

$$\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$$

- The two-port is **unconditionally stable** if:
- two conditions are simultaneously satisfied:
  - K > 1
  - |Δ| < 1
- together with the implicit conditions:
  - |S11| < 1
  - |S22| < 1

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|} > 1$$

$$|\Delta| = |S_{11} \cdot S_{22} - S_{12} \cdot S_{21}| < 1$$

μ Criterion

- Rollet's condition cannot be used to compare the relative stability of two or more devices because it involves constraints on two separate parameters, K and Δ

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-\Delta\cdot S_{11}^*|+|S_{12}\cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
  - $\mu > 1$
- together with the implicit conditions:
  - $|S_{11}| < 1$
  - $|S_{22}| < 1$
- In addition, it can be said that larger values of μ imply greater stability
  - μ is the distance from the center of the Smith Chart to the closest output stability circle

μ' Criterion

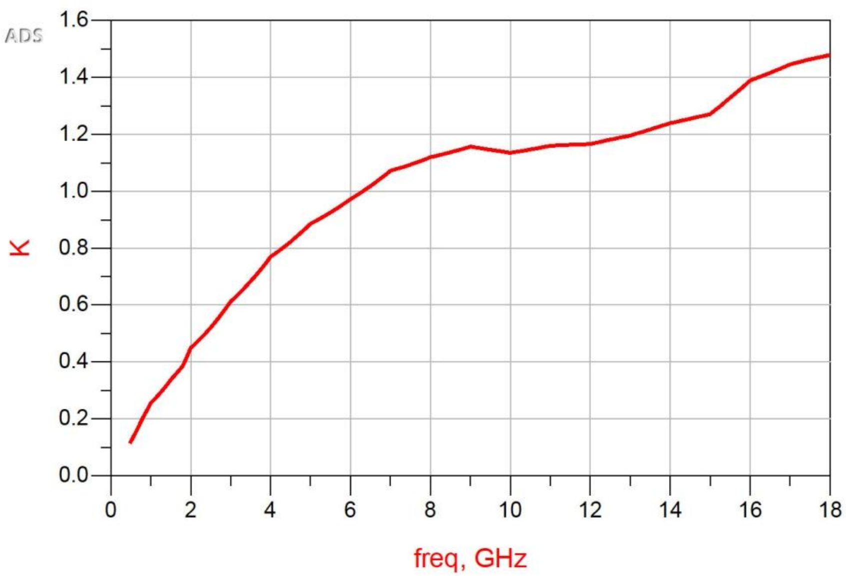
- Dual parameter to μ, determined in relation to the input stability circles

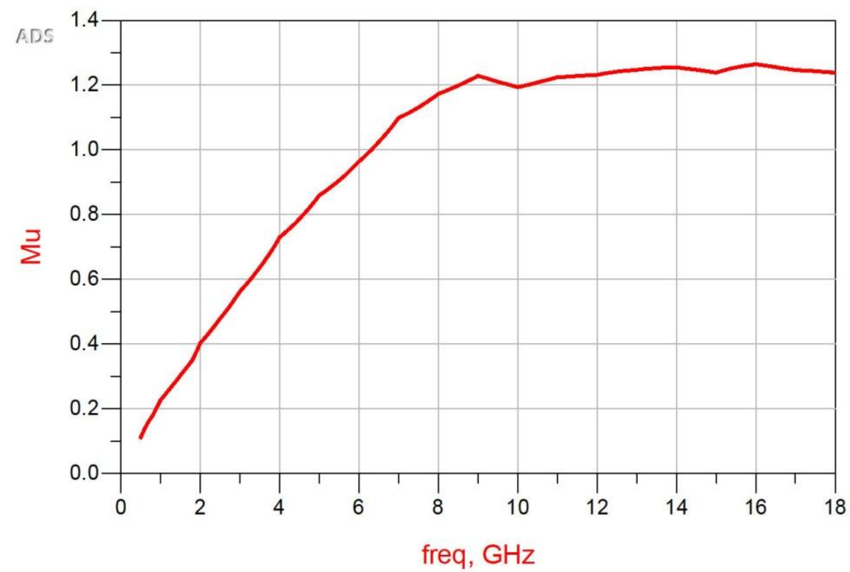
$$\mu' = \frac{1-|S_{22}|^2}{|S_{11}-\Delta\cdot S_{22}^*|+|S_{12}\cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
  - $\mu' > 1$
- together with the implicit conditions:
  - $|S_{11}| < 1$
  - $|S_{22}| < 1$
- In addition, it can be said that larger values of μ' imply greater stability
  - μ' is the distance from the center of the Smith Chart to the closest input stability circle

Stability

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz
- unconditionally stable for f > 6.31GHz

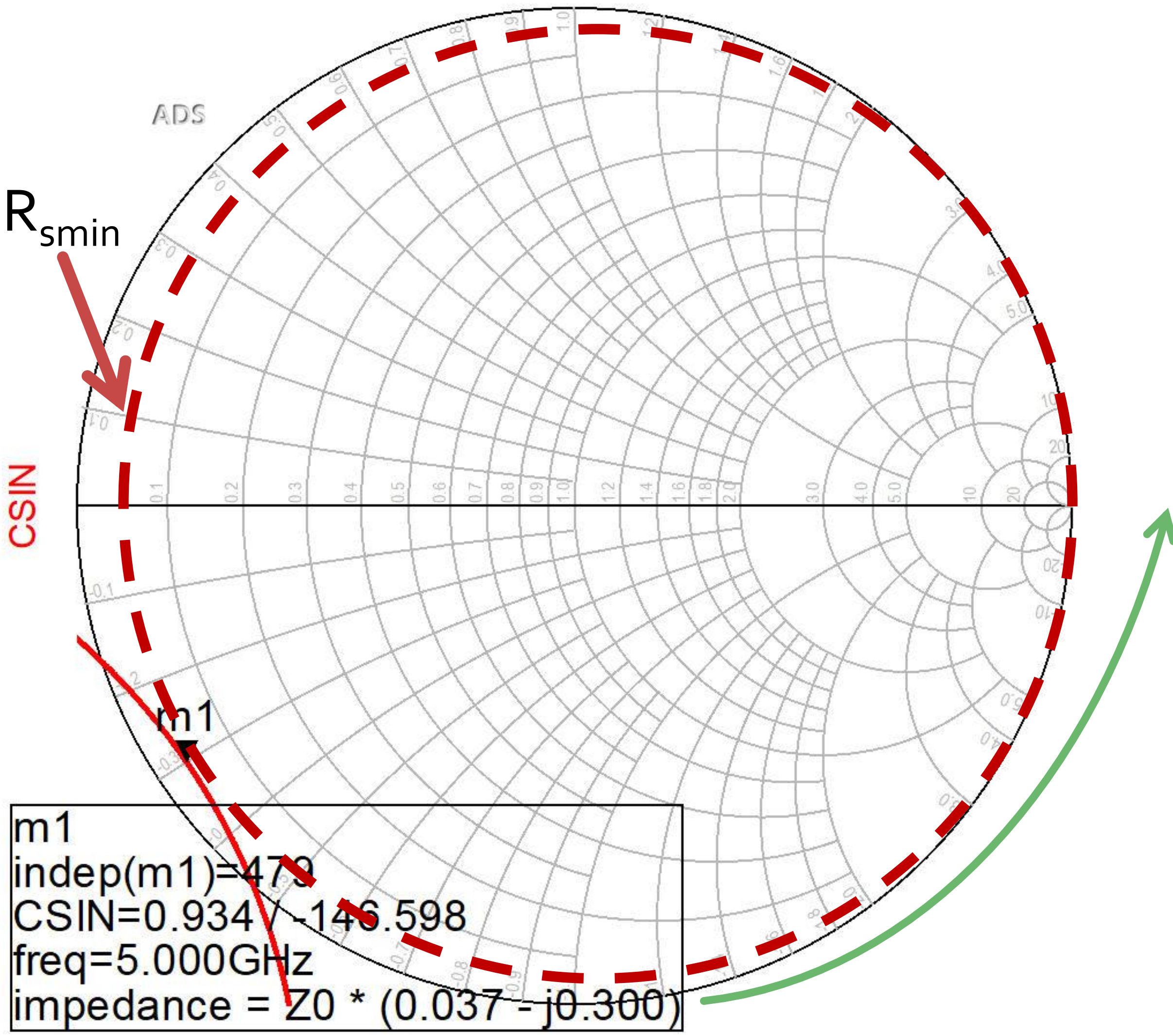


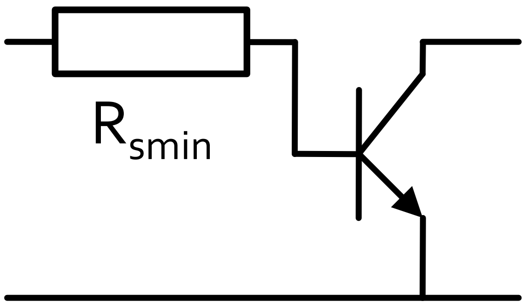


Stabilization of two-port

- Unconditional stability in a wide frequency range has some important advantages
  - Ex: We can use ATF 34143 to design a (conditionally) stable amplifier at 5GHz, but this design is useless if the amplifier oscillates at 500MHz ($\mu \approx 0.1$)
- **The minimal requirement** when working with conditionally stable devices is to **check stability** at several frequencies over the operating bandwidth and outside the bandwidth
- Unconditional stability can be forced by inserting series/shunt resistors at two-port's input/output (with loss of gain!)

Input series resistor

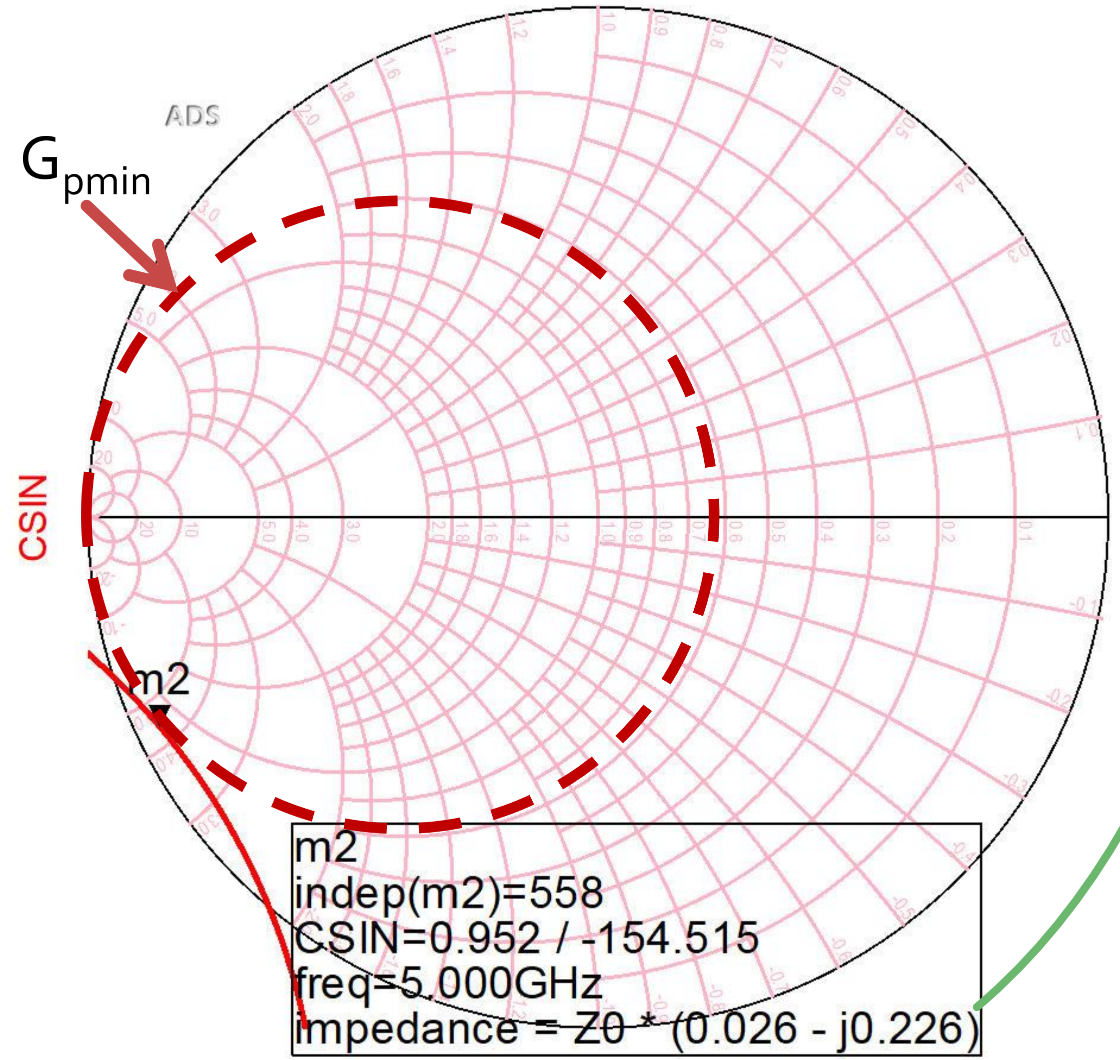


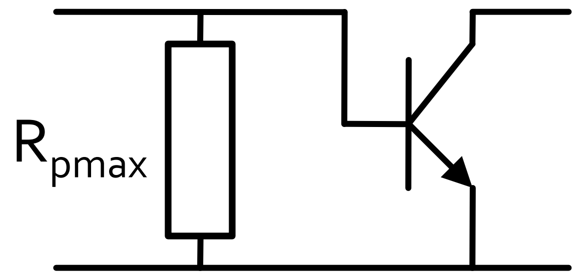


$$z = 0.037 - j \cdot 0.3$$

$$R_{s\min} = 0.037 \cdot 50\Omega = 1.85\Omega$$

Input shunt resistor





$$R_{p\max} = \frac{1}{G_{p\min}}$$

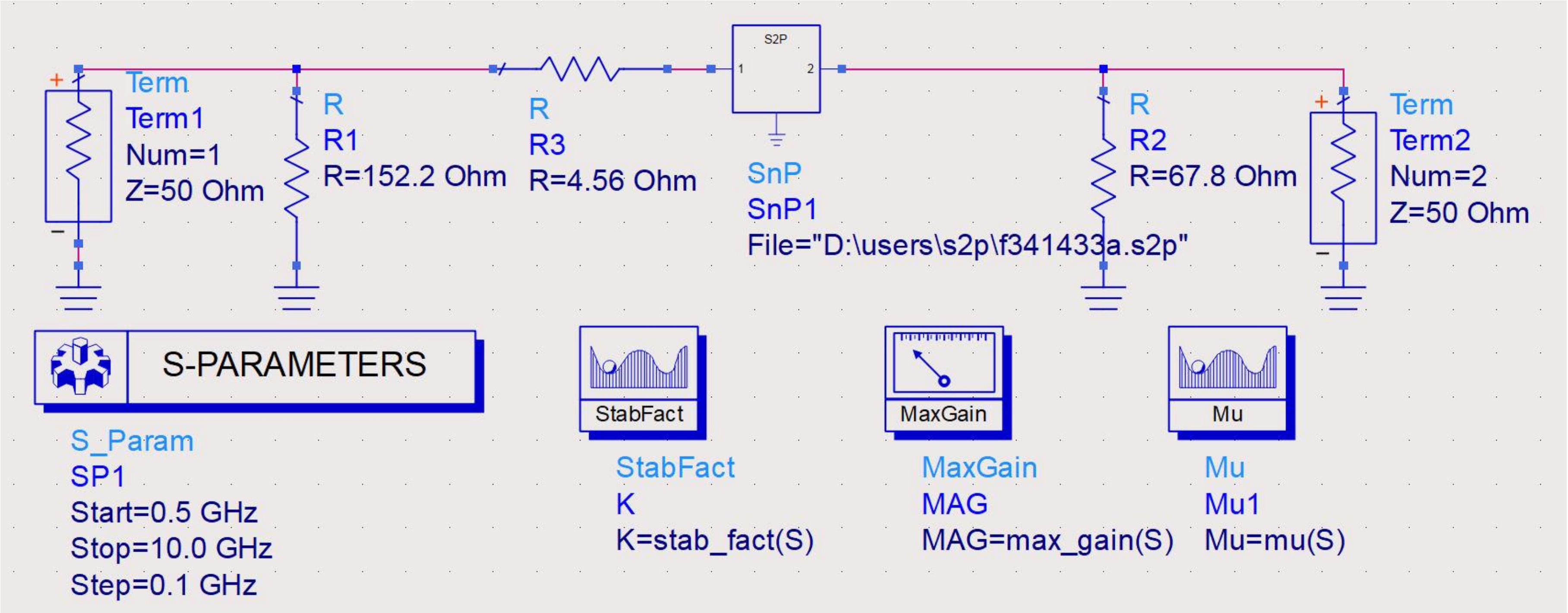
$$z = 0.026 - j \cdot 0.226$$

$$y = \frac{1}{z} = \frac{1}{0.026 - j \cdot 0.226}$$

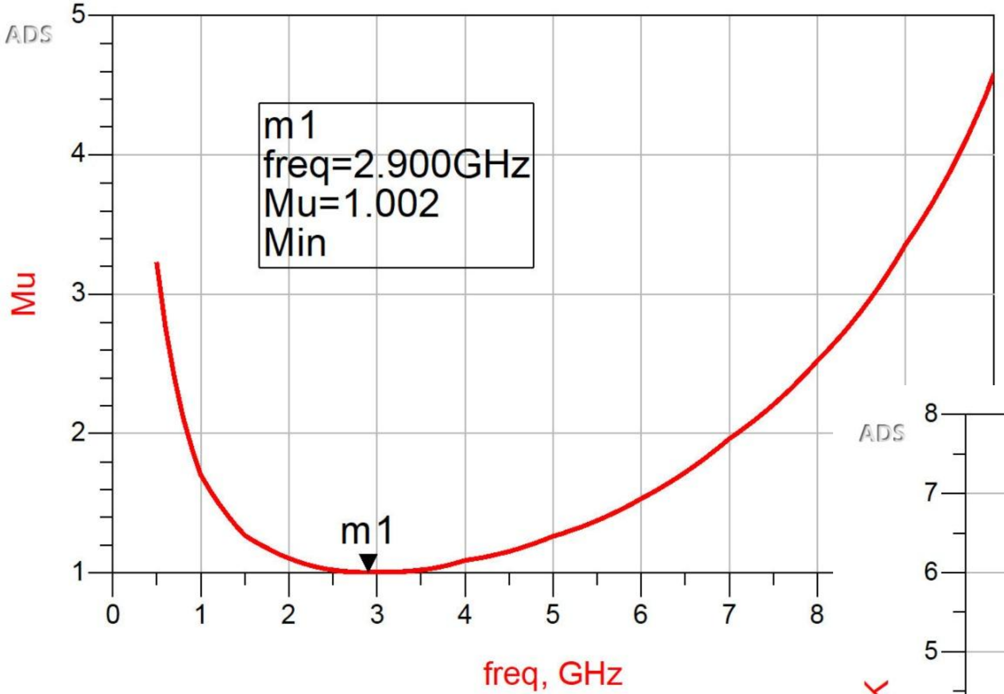
$$y = 0.502 + j \cdot 4.367$$

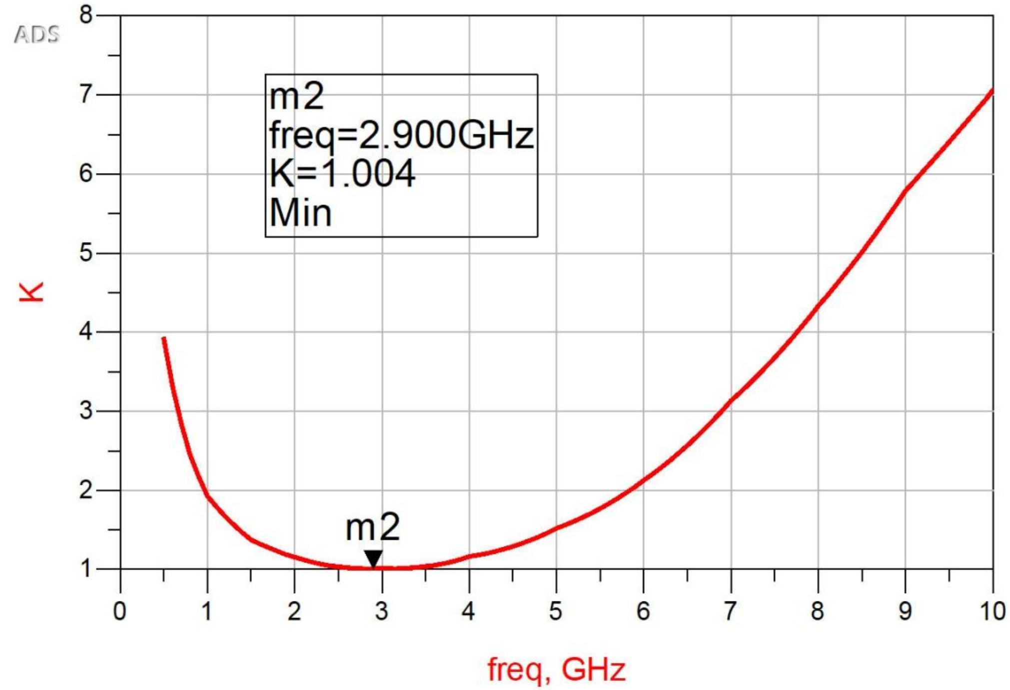
$$R_{p\max} = \frac{50\Omega}{0.502} = 99.6\Omega$$

Stabilization of two-port

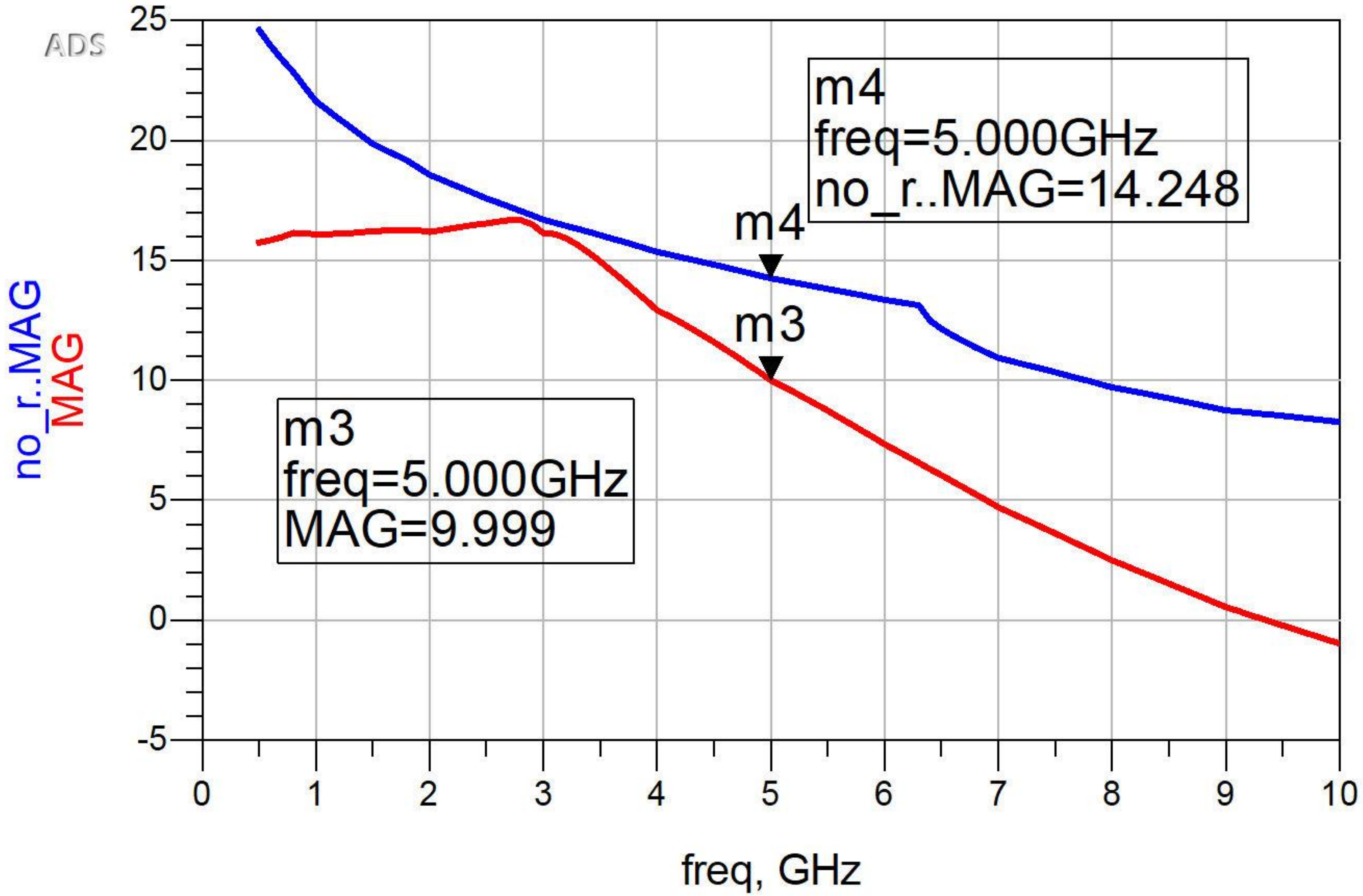


Stabilization of two-port





Stabilization of two-port

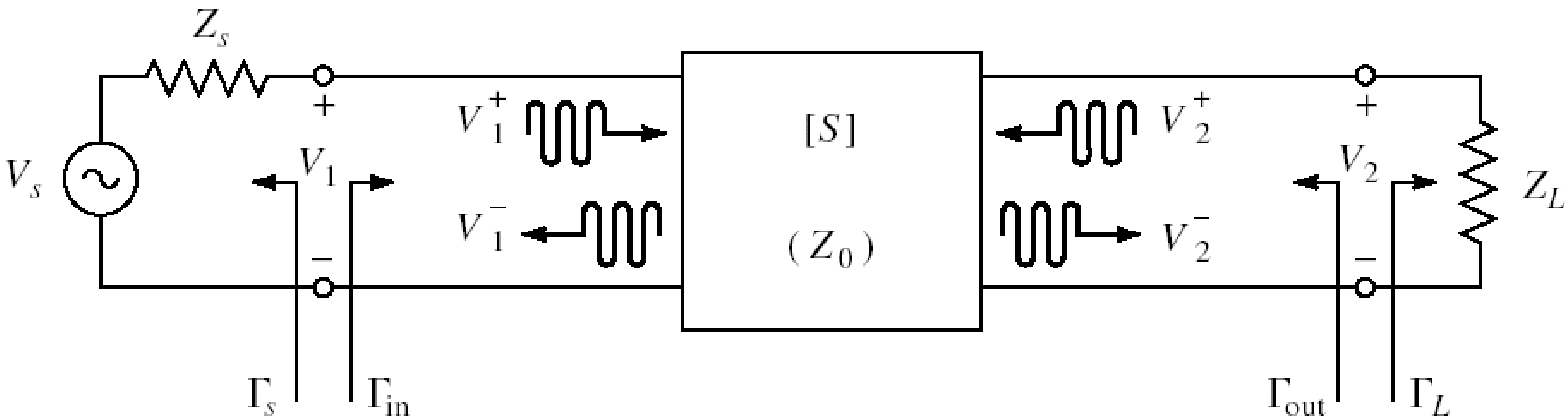


Continue

Microwave Amplifiers

Power Gain of Microwave Amplifiers

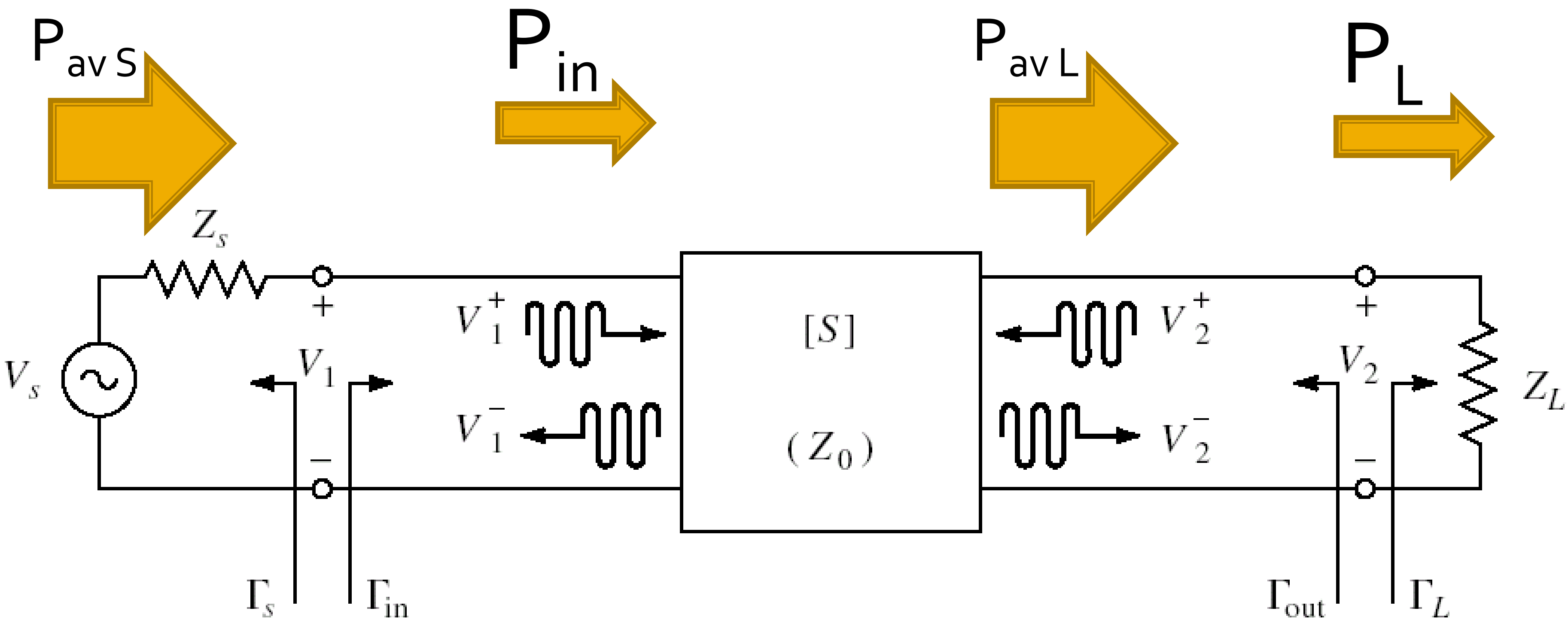
Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - **power gain**
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Power / Matching

- Two ports in which matching influences the power transfer



Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av\,L}}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)}{|1-S_{22}\cdot\Gamma_L|^2 \cdot \left(1-|\Gamma_{out}|^2\right)}$$

- **Transducer** power gain

$$G_T = \frac{P_L}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)\cdot\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\cdot\Gamma_{in}|^2 \cdot |1-S_{22}\cdot\Gamma_L|^2}$$

$$\Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

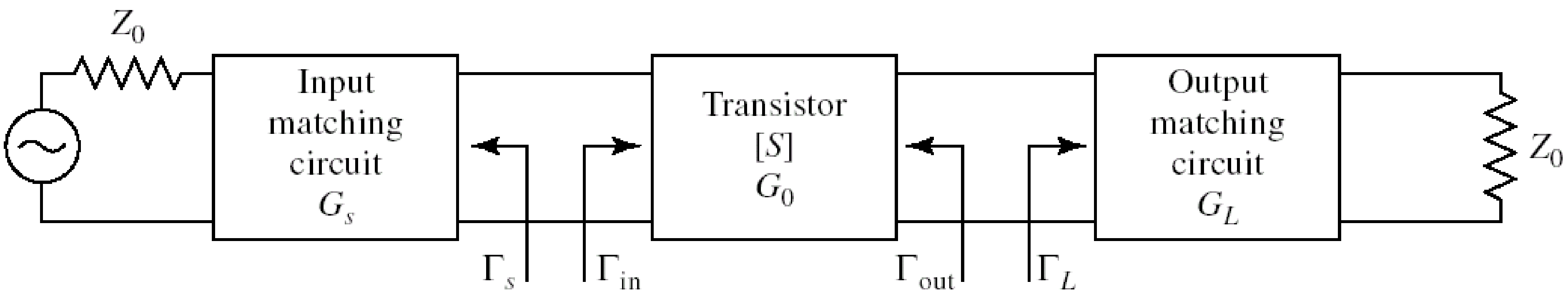
- **Unilateral transducer** power gain

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$S_{12} \cong 0 \qquad \Gamma_{in} = S_{11}$$

Input and output can be treated independently

Design for Maximum Gain



- Maximum power gain (complex conjugate matching):

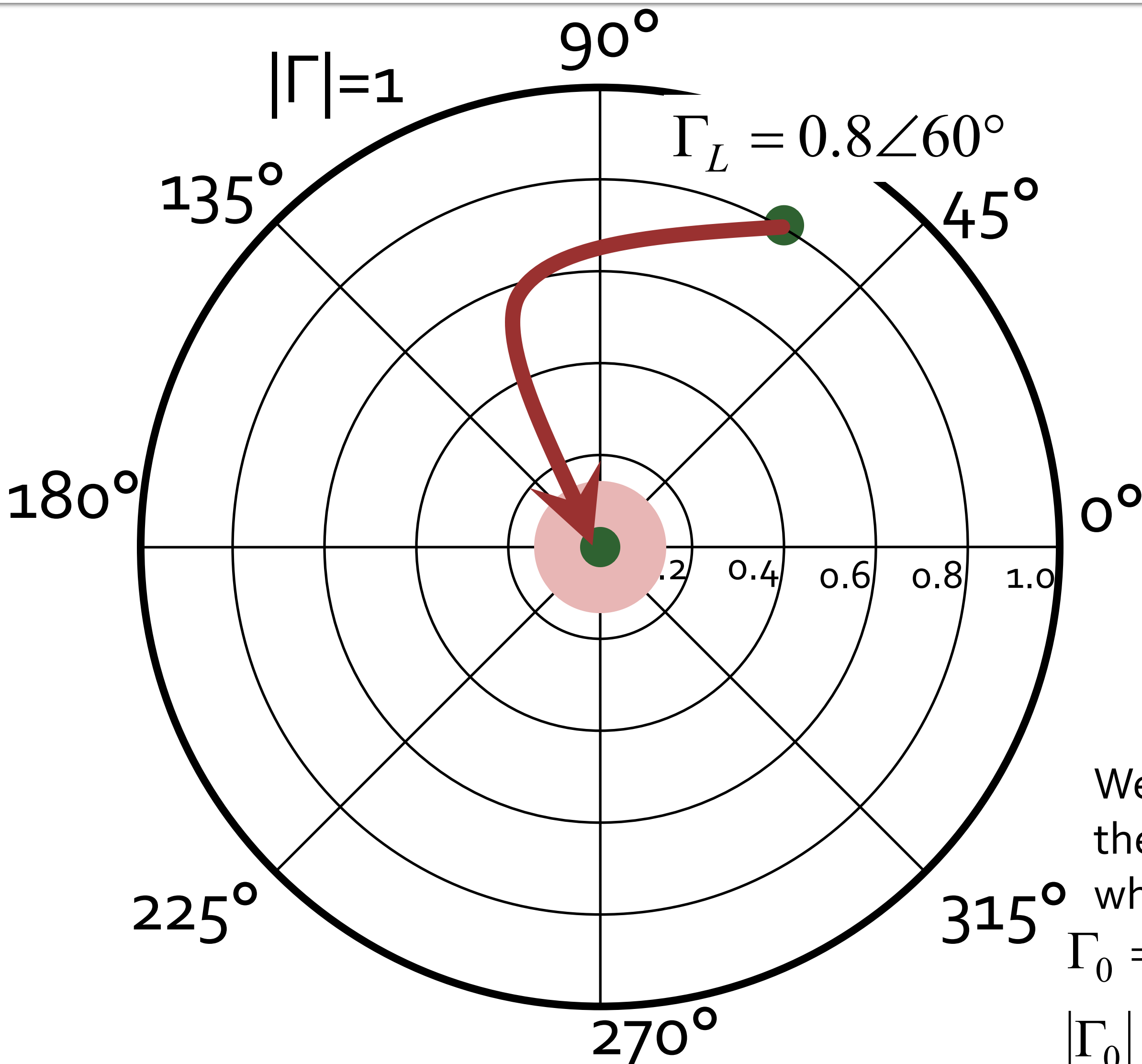
$$\Gamma_{in} = \Gamma_S^* \qquad \Gamma_{out} = \Gamma_L^*$$

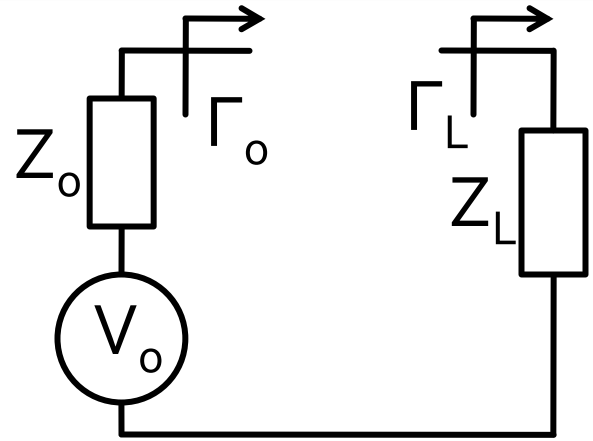
- For lossless matching sections

$$G_{T\max} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right) \cdot \left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S \cdot \Gamma_{in}|^2 \cdot |1-S_{22} \cdot \Gamma_L|^2} \qquad G_{T\max} = \frac{1}{1-|\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22} \cdot \Gamma_L|^2}$$

- For the general case of the bilateral transistor (S12≠0) Γ_{in} and Γ_{out} depend on each other so the input and output sections must be matched simultaneously

The Smith Chart, matching, $Z_L \neq Z_o$





Matching Z_L load to Z_o source.
We normalize Z_L over Z_o

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

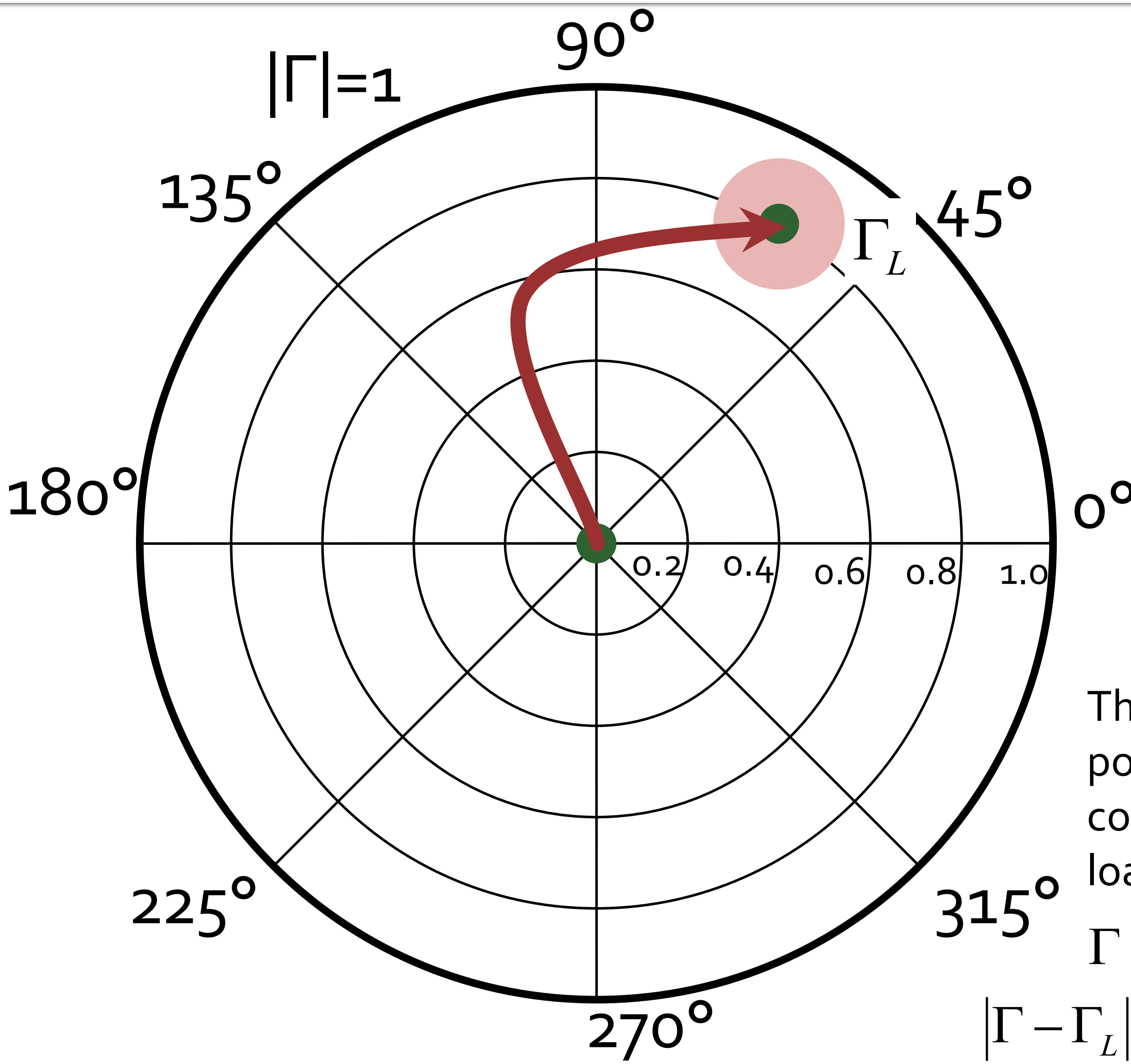
$$\Gamma_L = 0.8\angle 60°$$

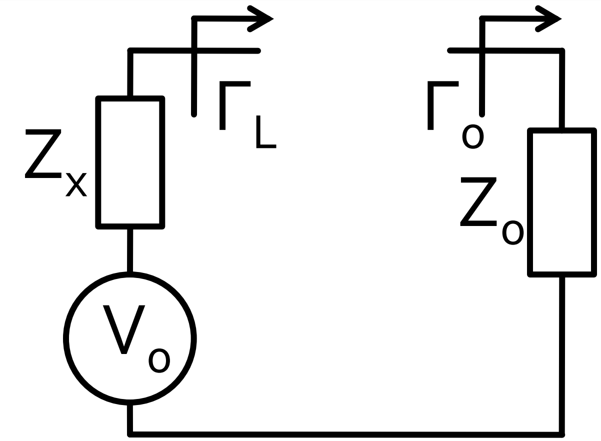
We must move the point denoting the reflection coefficient in the area where with a Z_o source we have:

$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \leq \Gamma_m$ “good enough” match

The Smith Chart, matching, $Z_L=Z_o$





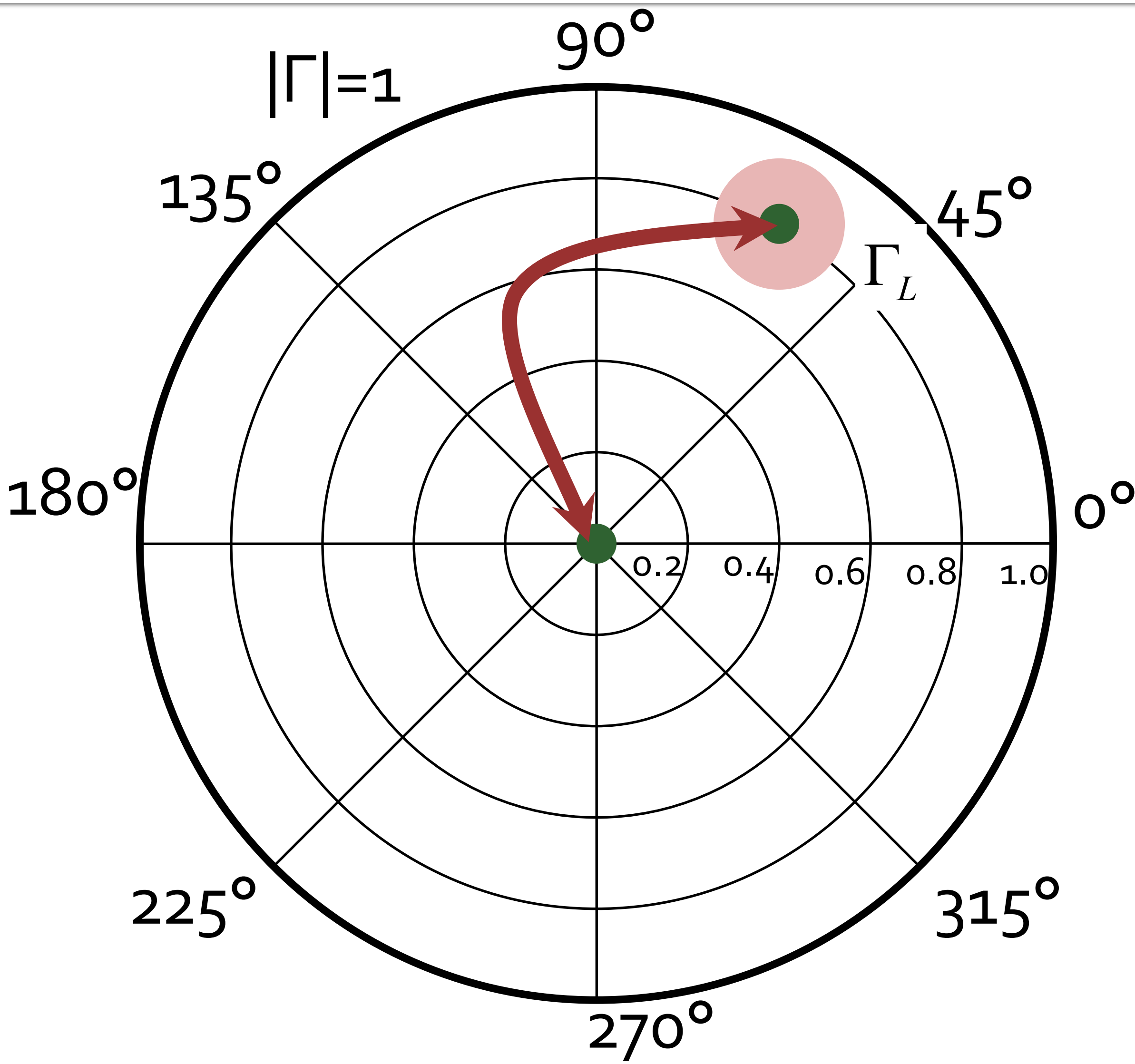
The source (eg. the transistor) having Z_X needs to see a certain reflection coefficient Γ_L towards the load Z_o

The matching circuit must move the point denoting the reflection coefficient in the area where for a Z_o load (Γ_o=0) we see towards it:

$\Gamma = \Gamma_L$ perfect match

$|\Gamma - \Gamma_L| \leq \Gamma_m$ "good enough" match

The Smith Chart, matching , $Z_L \neq Z_o$, $Z_L = Z_o$



- The matching sections needed to move
  - Γ_L in Γ_o
  - Γ_o in Γ_L
- are **identical**. They differ only by the **order** in which the elements are introduced into the matching circuit
- As a result, we can use in match design the same:
  - **methods**
  - **formulae**

Simultaneous matching

$$\Gamma_{in} = \Gamma_S^*$$

$$\Gamma_{in} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_S^* = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \Gamma_L^*$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

- We find Γ_S

$$\Gamma_S = S_{11}^* + \frac{S_{12}^* \cdot S_{21}^*}{1/\Gamma_L^* - S_{22}^*}$$

$$\Gamma_L^* = \frac{S_{22} - \Delta \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

$$\Gamma_S \cdot \left(1 - |S_{22}|^2\right) + \Gamma_S^2 \cdot \left(\Delta \cdot S_{22}^* - S_{11}\right) = \Gamma_S \cdot \left(\Delta \cdot S_{11}^* \cdot S_{22}^* - |S_{22}|^2 - \Delta \cdot S_{12}^* \cdot S_{21}^*\right) + S_{11}^* \cdot \left(1 - |S_{22}|^2\right) + S_{12}^* \cdot S_{21}^* \cdot S_{22}$$

Simultaneous matching

$$\Delta \cdot \left(S_{11}^{*} \cdot S_{22}^{*} - S_{12}^{*} \cdot S_{21}^{*}\right) = |\Delta|^{2}$$

$$\Gamma_S^{2} \cdot \underbrace{\left(S_{11} - \Delta \cdot S_{22}^{*}\right)}_{C} + \Gamma_S \cdot \underbrace{\left(|\Delta|^{2} - |S_{11}|^{2} + |S_{22}|^{2} - 1\right)}_{-B} + \underbrace{\left(S_{11}^{*} - \Delta^{*} \cdot S_{22}\right)}_{C^{*}} = 0$$

- A quadratic equation

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^{2} - 4 \cdot |C_1|^{2}}}{2 \cdot C_1}$$

- Similarly

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^{2} - 4 \cdot |C_2|^{2}}}{2 \cdot C_2}$$

- With variables defined as:

$$\begin{cases} B_1 = 1 + |S_{11}|^{2} - |S_{22}|^{2} - |\Delta|^{2} \\ C_1 = S_{11} - \Delta \cdot S_{22}^{*} \end{cases} \quad \begin{cases} B_2 = 1 + |S_{22}|^{2} - |S_{11}|^{2} - |\Delta|^{2} \\ C_2 = S_{22} - \Delta \cdot S_{11}^{*} \end{cases}$$

Simultaneous matching

- Simultaneous matching is possible if:

$$B_1^2 - 4\cdot|C_1|^2 > 0 \qquad B_2^2 - 4\cdot|C_2|^2 > 0$$

$$\Delta\cdot\left(S_{11}^*\cdot S_{22}^* - S_{12}^*\cdot S_{21}^*\right) = |\Delta|^2$$

$$|C_1|^2 = \left|S_{11} - \Delta\cdot S_{22}^*\right|^2 = |S_{12}|^2\cdot|S_{21}|^2 + \left(1-|S_{22}|^2\right)\cdot\left(|S_{11}|^2 - |\Delta|^2\right)$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1+|S_{11}|^2\right)^2 + \left(|S_{22}|^2 + |\Delta|^2\right)^2 -$$
$$-2\cdot\left(1+|S_{11}|^2\right)\cdot\left(|S_{22}|^2+|\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2 - 4\cdot\left(1-|S_{22}|^2\right)\cdot\left(|S_{22}|^2-|\Delta|^2\right)$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1+|S_{11}|^2\right)^2 + \left(|S_{22}|^2 + |\Delta|^2\right)^2 -$$
$$-4\cdot|S_{11}|^2 - 4\cdot|S_{22}|^2\cdot|\Delta|^2 - 2\cdot\left(1-|S_{11}|^2\right)\cdot\left(|S_{22}|^2-|\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2$$

Simultaneous matching

$$B_1^2 - 4\cdot|C_1|^2 = \left(1+|S_{11}|^2\right)^2 + \left(|S_{22}|^2+|\Delta|^2\right)^2 - \\ -4\cdot|S_{11}|^2 - 4\cdot|S_{22}|^2\cdot|\Delta|^2 - 2\cdot\left(1-|S_{11}|^2\right)\cdot\left(|S_{22}|^2-|\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1-|S_{11}|^2\right)^2 + \left(|S_{22}|^2-|\Delta|^2\right)^2 - 2\cdot\left(1-|S_{11}|^2\right)\cdot\left(|S_{22}|^2-|\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2\right)^2 - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(K\cdot 2\cdot|S_{12}\cdot S_{21}|\right)^2 - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2-1\right)$$

- Similarly

$$B_2^2 - 4\cdot|C_2|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2-1\right)$$

Simultaneous matching

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4\cdot|C_1|^2}}{2\cdot C_1} \qquad \Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4\cdot|C_2|^2}}{2\cdot C_2}$$

- Solutions must ensure stability:

$$|\Gamma_S| < 1 \qquad |\Gamma_L| < 1$$

$$|\Delta| = |S_{11}\cdot S_{22} - S_{12}\cdot S_{21}| < 1 \qquad \begin{cases} B_1 > 0 \\ B_2 > 0 \end{cases}$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\cdot|S_{12}\cdot S_{21}|} > 1 \qquad \begin{cases} B_1^2 - 4\cdot|C_1|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2 - 1\right) > 0 \\ B_2^2 - 4\cdot|C_2|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2 - 1\right) > 0 \end{cases}$$

Simultaneous matching

- Simultaneous matching can be achieved **if and only if** the amplifier is **unconditionally stable** at the operating frequency, and $|\Gamma|<1$ solutions are those with "–" sign of quadratic solutions

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4\cdot|C_1|^2}}{2\cdot C_1} \qquad \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4\cdot|C_2|^2}}{2\cdot C_2}$$

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta\cdot S_{22}^* \end{cases} \qquad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta\cdot S_{11}^* \end{cases}$$

Simultaneous matching

- In the case of the simultaneous matching the amplifier achieves the maximum transducer power gain for the bilateral transistor

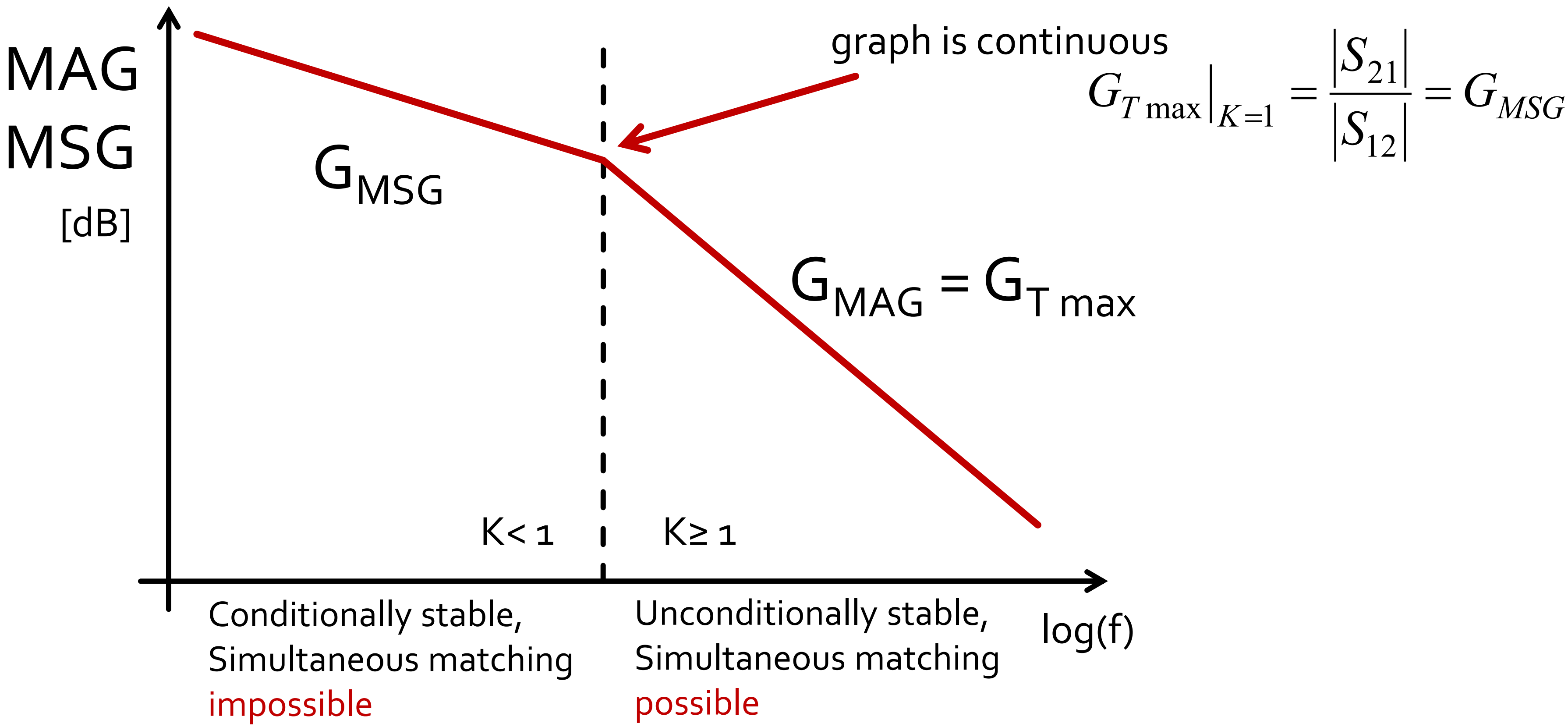
$$G_{T\max} = \frac{|S_{21}|}{|S_{12}|} \cdot \left(K - \sqrt{K^2 - 1}\right)$$

- If the device is **not unconditionally stable** at a certain frequency, we can use MSG (Maximum Stable Gain) as an indicator of the capability to obtain a power gain in stable conditions

$$G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$$

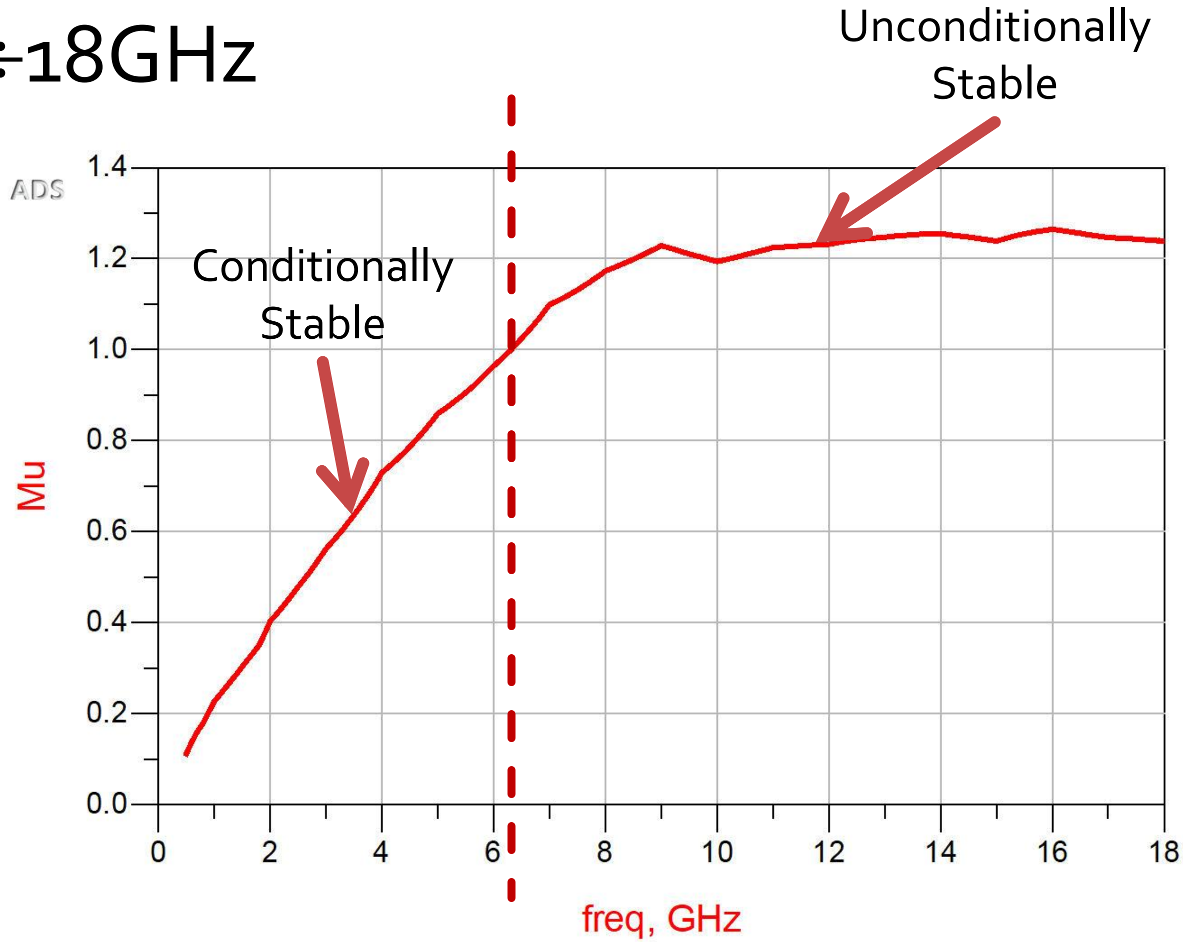
Maximum Available Gain

- Indicator across full frequency range of the capability to obtain a power gain



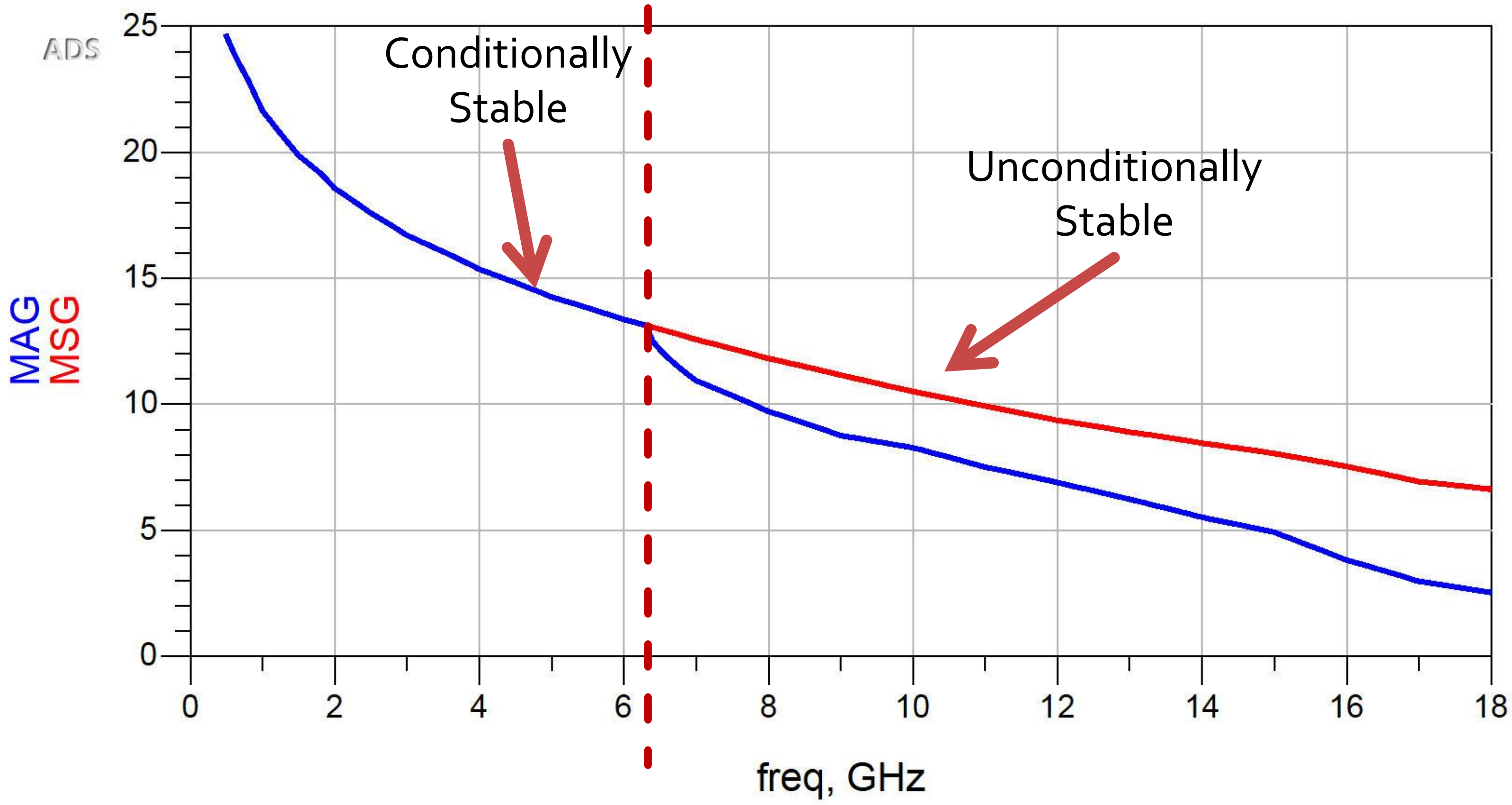
Stability

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



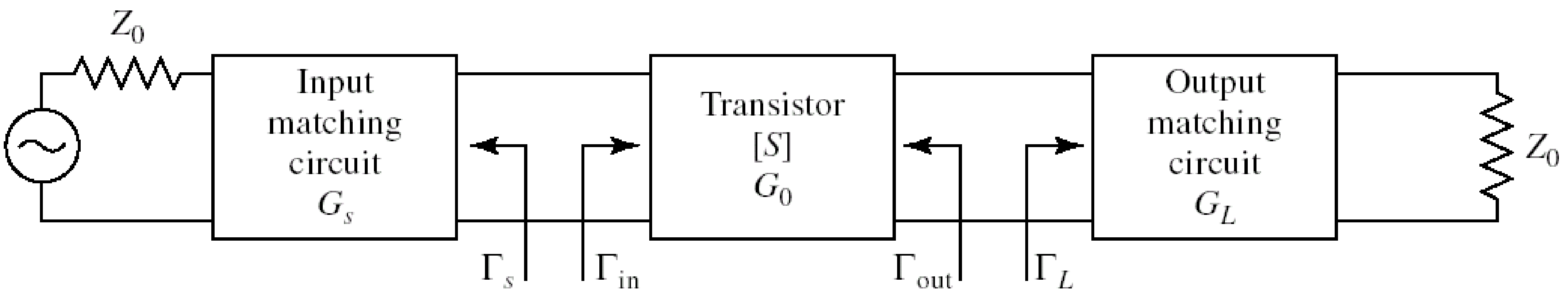
MAG/MSG

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



Simultaneous matching, unilateral transistor

- In the case of **unilateral** amplifier/transistor (S12 = 0) simultaneous matching implies:



$$\Gamma_{in} = S_{11} \qquad \Gamma_{out} = S_{22}$$

$$\Gamma_S = S_{11}^* \qquad \Gamma_L = S_{22}^*$$

$$G_{T\max} = \frac{1}{1-|\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2} \qquad G_{TU\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2}$$

Example

- ATF-34143 **at Vds=3V Id=20mA**.
  - without stabilization K = **0.886**, MAG = 14.248dB @ 5GHz
  - cannot be used with this bias conditions
- ATF-34143 **at Vds=4V Id=40mA**
  - without stabilization K = **1.031**, MAG = 12.9dB @ 5GHz
  - we use this bias conditions for simultaneous matching

Example

- ATF-34143 **at Vds=4V Id=40mA**.
- @5GHz
  - $S_{11} = 0.64\angle 111°$
  - $S_{12} = 0.117\angle -27°$
  - $S_{21} = 2.923\angle -6°$
  - $S_{22} = 0.21\angle 111°$

Computations

- Complex S Parameters
  - S11 = -0.229+0.597·j
  - S12 = 0.104-0.053·j
  - S21 = 2.907-0.306·j
  - S22 = -0.075+0.196·j

$$\begin{cases} S_{11} = 0.64\angle 111^\circ \\ S_{11} = 0.64 \cdot \cos 111^\circ + j \cdot 0.64 \cdot \sin 111^\circ \end{cases}$$

$$G_{T\max} = \frac{|S_{21}|}{|S_{12}|} \cdot \left(K - \sqrt{K^2 - 1}\right) = 19.497 = 12.9\,\text{dB}$$

$$G_{TU\max} = \frac{1}{1 - |S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |S_{22}|^2} = 15.139 = 11.8\,\text{dB}$$

Computations

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases} \qquad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

$$\begin{cases} B_1 = ? \\ C_1 = ? \end{cases} \qquad \begin{cases} B_2 = ? \\ C_2 = ? \end{cases}$$

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1} \qquad \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

$$\Gamma_S = ? \qquad \Gamma_L = ?$$

Computations

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases} \qquad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

$$\begin{cases} B_1 = 1.207 \\ C_1 = -0.277 + j \cdot 0.529 \end{cases} \qquad \begin{cases} B_2 = 0.476 \\ C_2 = -0.222 - j \cdot 0.013 \end{cases}$$

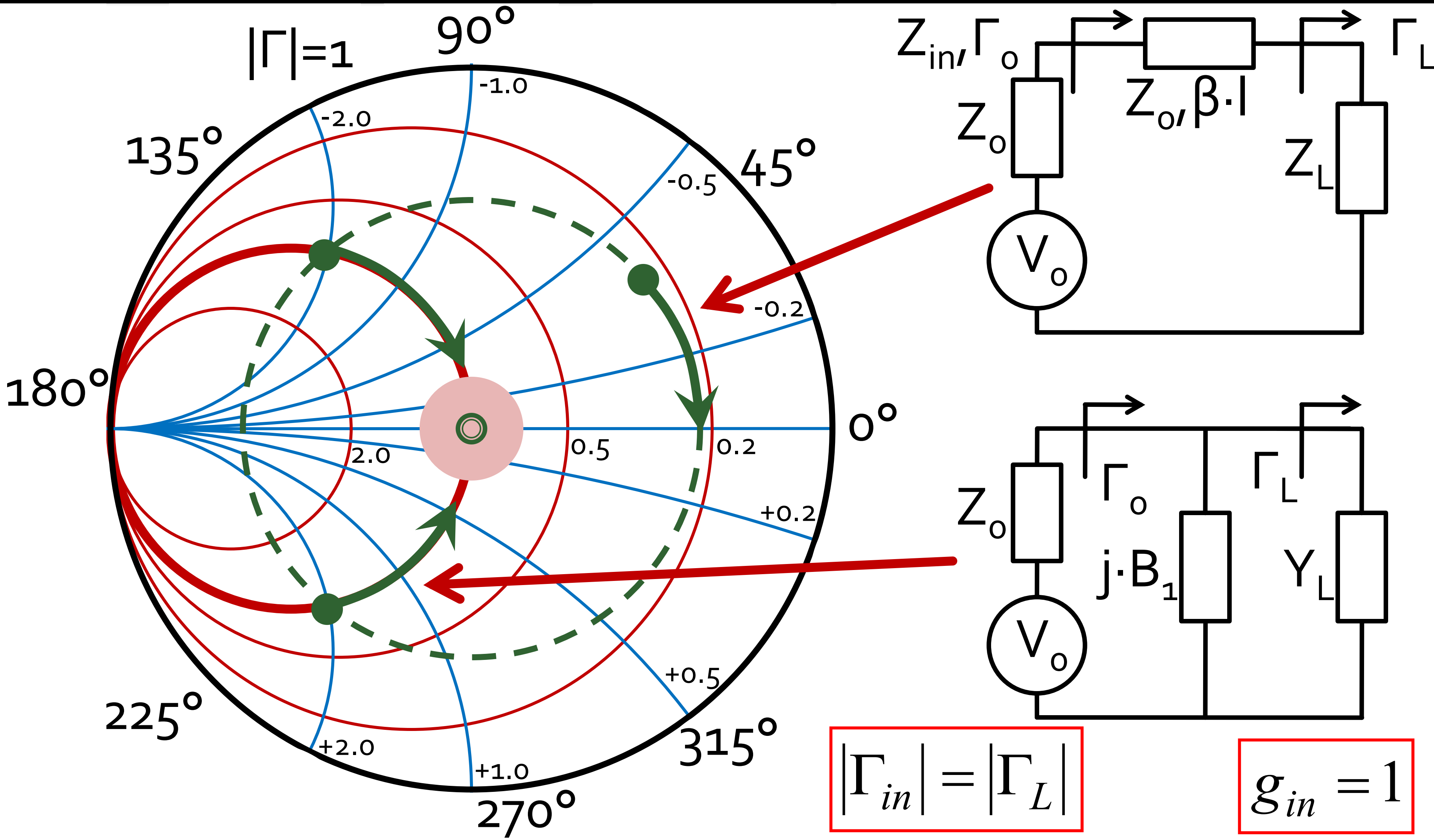
$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1} \qquad \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

$$\Gamma_S = -0.403 - j \cdot 0.768 \qquad \Gamma_L = -0.685 + j \cdot 0.04$$

$$|\Gamma_S| = 0.867 < 1 \qquad |\Gamma_L| = 0.686 < 1$$

$$\Gamma_S = 0.867 \angle -117.7^\circ \qquad \Gamma_L = 0.686 \angle 176.7^\circ$$

Shunt stub matching, L8



$|\Gamma_{in}| = |\Gamma_L|$

$g_{in} = 1$

Analytical solution (Γ_S)

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.867 \angle -117.7^\circ$$

$$|\Gamma_S| = 0.867; \quad \varphi = -117.7^\circ \qquad \cos(\varphi + 2\theta) = -0.867 \Rightarrow \quad (\varphi + 2\theta) = \pm 150.1^\circ$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

$$(-117.7^\circ + 2\theta) = +150.1^\circ \qquad \theta = 133.9^\circ \qquad \operatorname{Im} y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -3.477$$

$$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = -74^\circ (+180^\circ) \rightarrow \theta_{sp} = 106^\circ$$

  - **"-" solution**

$$(-117.7^\circ + 2\theta) = -150.1^\circ \qquad \theta = -16.2^\circ (+180^\circ) \rightarrow \theta = 163.8^\circ$$

$$\operatorname{Im} y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +3.477 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = 74^\circ$$

Analytical solution (Γ_L)

$$\cos(\varphi + 2\theta) = -|\Gamma_L|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}}$$

$$\Gamma_L = 0.686\angle 176.7°$$

$$|\Gamma_L| = 0.686; \quad \varphi = 176.7°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**
  - **"-" solution**

Analytical solution (Γ_L)

$$\cos(\varphi + 2\theta) = -|\Gamma_L|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}}$$

$$\Gamma_L = 0.686\angle 176.7°$$

$$|\Gamma_L| = 0.686; \quad \varphi = 176.7° \qquad \cos(\varphi + 2\theta) = -0.686 \Rightarrow \quad (\varphi + 2\theta) = \pm 133.3°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

$$(176.7° + 2\theta) = +133.3° \qquad \theta = -21.7°(+180°) \rightarrow \theta = 158.3°$$

$$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_L) = -62.1°(+180°) \rightarrow \theta_{sp} = 117.9° \qquad \operatorname{Im} y_L = \frac{-2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}} = -1.885$$

  - **"-" solution**

$$(176.7° + 2\theta) = -133.3° \qquad \theta = -155°(+180°) \rightarrow \theta = 25°$$

$$\operatorname{Im} y_L = \frac{+2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}} = +1.885 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_L) = 62.1°$$

Complete analytical solution

- We choose **one** of the two possible solutions for the input matching

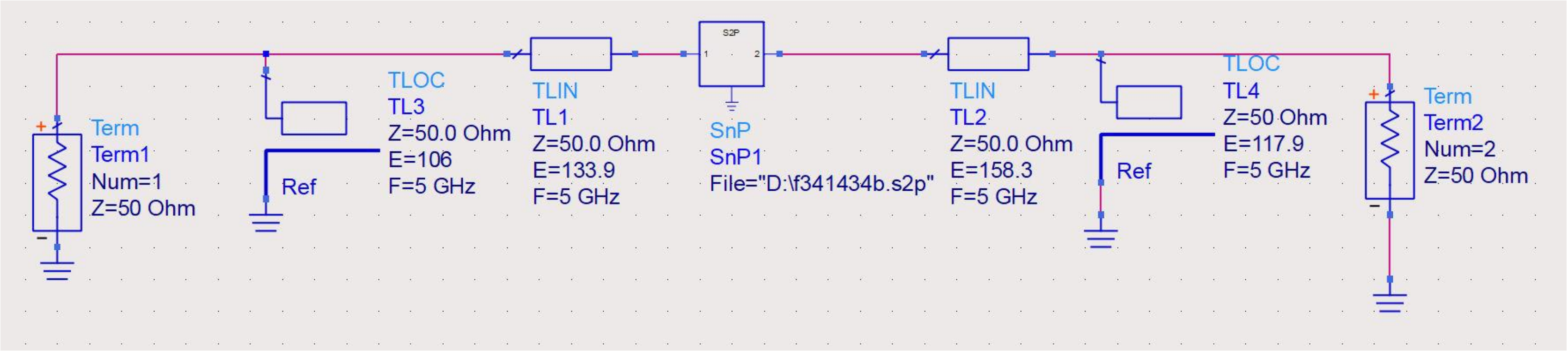
$$(\varphi+2\theta)=\begin{cases}+150.1^\circ\\-150.1^\circ\end{cases}\quad \theta=\begin{cases}133.9^\circ\\163.8^\circ\end{cases}\quad \mathrm{Im}[y_S(\theta)]=\begin{cases}-3.477\\+3.477\end{cases}\quad \theta_{sp}=\begin{cases}-74^\circ+180^\circ=106^\circ\\+74^\circ\end{cases}$$

- Similarly for the output matching

$$(\varphi+2\theta)=\begin{cases}+133.3^\circ\\-133.3^\circ\end{cases}\quad \theta=\begin{cases}158.3^\circ\\25.0^\circ\end{cases}\quad \mathrm{Im}[y_S(\theta)]=\begin{cases}-1.885\\+1.885\end{cases}\quad \theta_{sp}=\begin{cases}117.9^\circ\\62.1^\circ\end{cases}$$

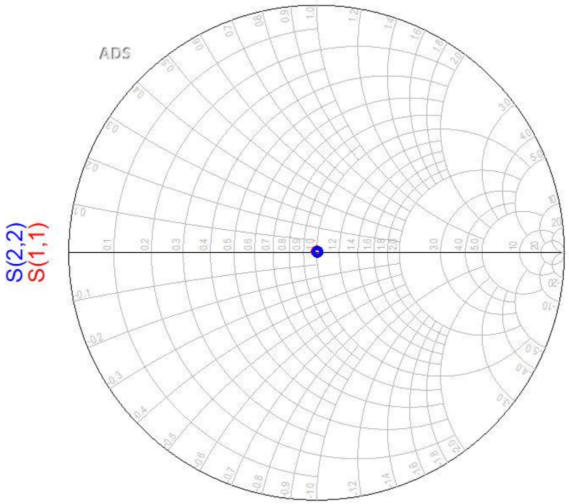
- In total there are **4** possible solutions input/output

ADS



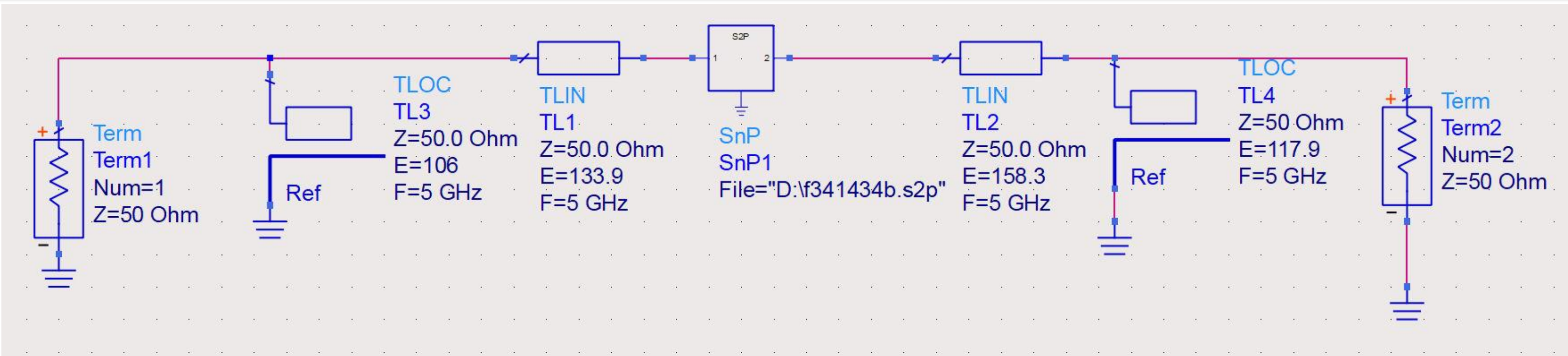
Eqn GT=10*log(mag(S(2,1))**2)

freq	S(2,1)	GT	S(1,1)	S(2,2)
5.000 GHz	4.415 / 157.353	12.900	0.004 / 86.088	0.004 / 37.766

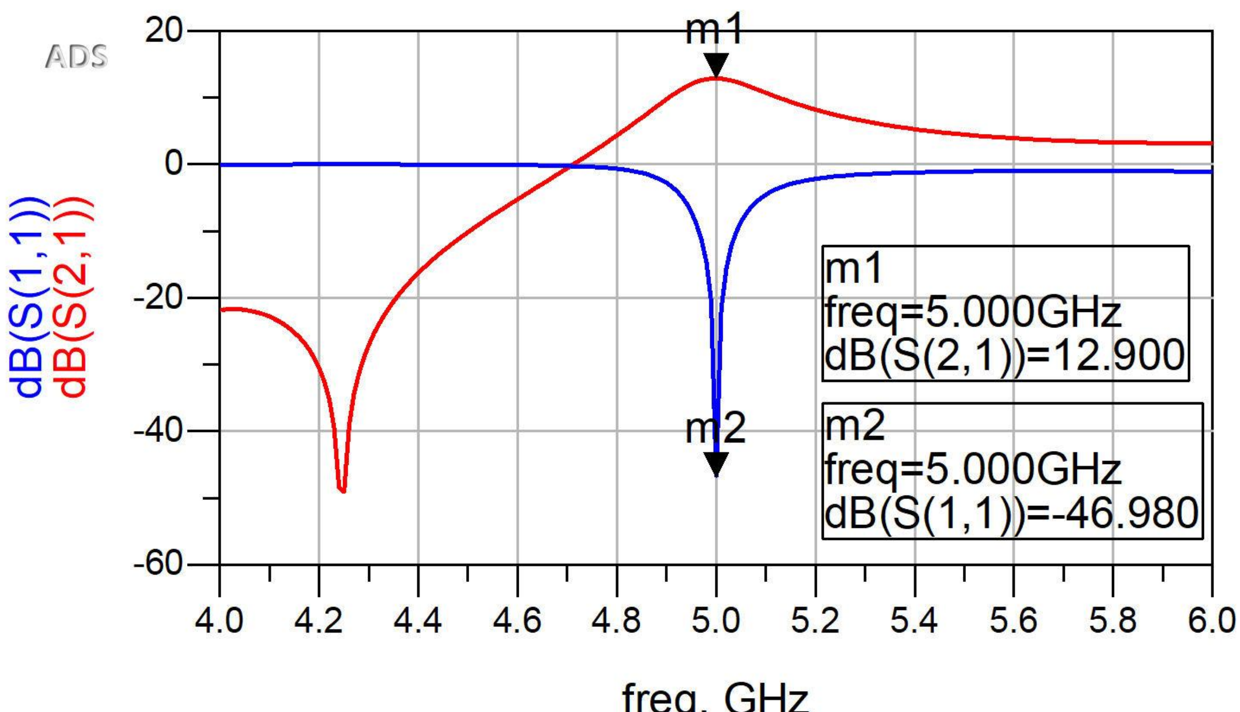


freq (5.000GHz to 5.000GHz)

ADS



Term
Term1
Num=1
Z=50 Ohm
Ref
TLOC
TL3
Z=50.0 Ohm
E=106
F=5 GHz
TLIN
TL1
Z=50.0 Ohm
E=133.9
F=5 GHz
S2P
SnP
SnP1
File="D:\f341434b.s2p"
TLIN
TL2
Z=50.0 Ohm
E=158.3
F=5 GHz
Ref
TLOC
TL4
Z=50 Ohm
E=117.9
F=5 GHz
Term
Term2
Num=2
Z=50 Ohm

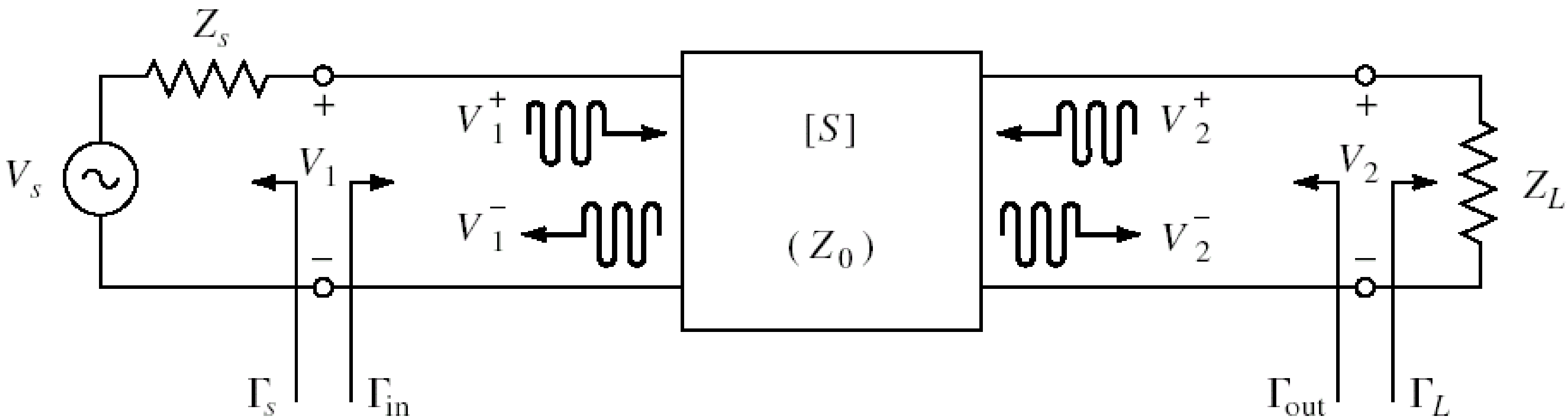


ADS
dB(S(1,1))
dB(S(2,1))
20
0
-20
-40
-60
4.0
4.2
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
freq, GHz
m1
m2
m1
freq=5.000GHz
dB(S(2,1))=12.900
m2
freq=5.000GHz
dB(S(1,1))=-46.980

Microwave Amplifiers

Design for Specified Gain

Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - **power gain**
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Design for Specified Gain

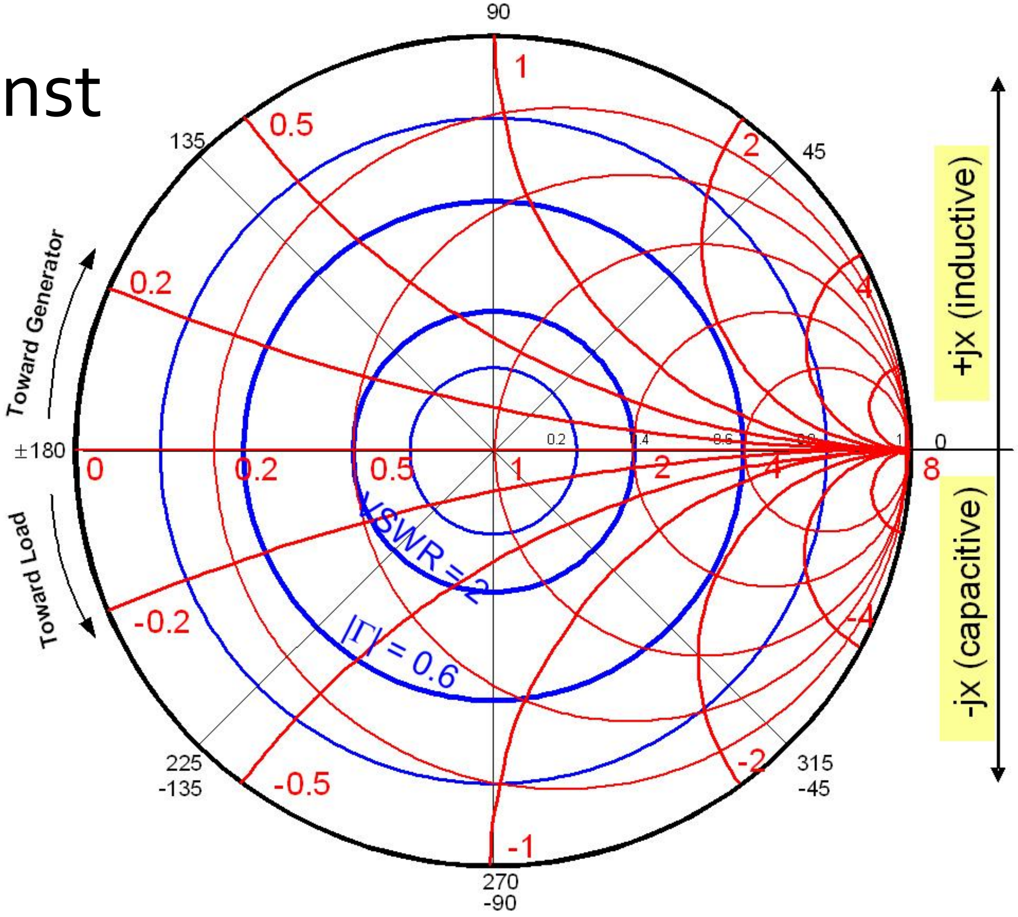
- In many cases we need an approach other than "brute force" when we prefer to design for **less than the maximum obtainable gain**, in order to:
  - improve noise behavior (Lab 3 + Lect. 10 next)
  - improve stability
  - improve VSWR
  - control performance at multiple frequencies
  - improve amplifier's bandwidth

Constant VSWR circles

- Certain applications may require a certain ratio between maximum / minimum line voltage

  - VSWR = const → Γ = const

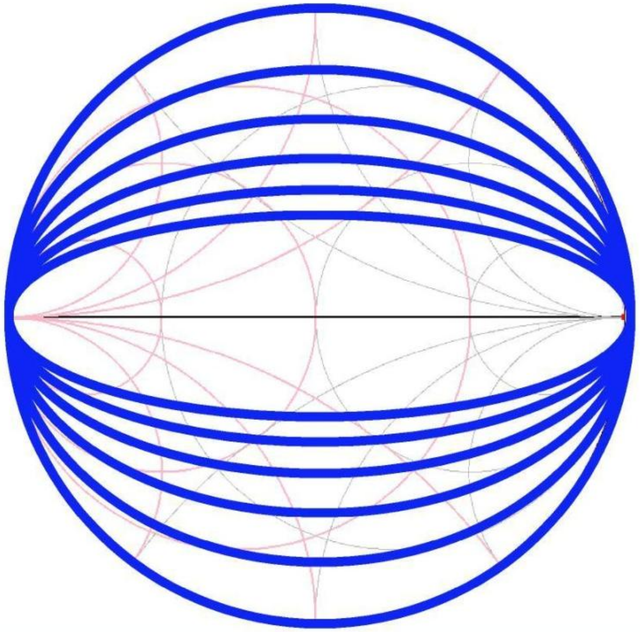
$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

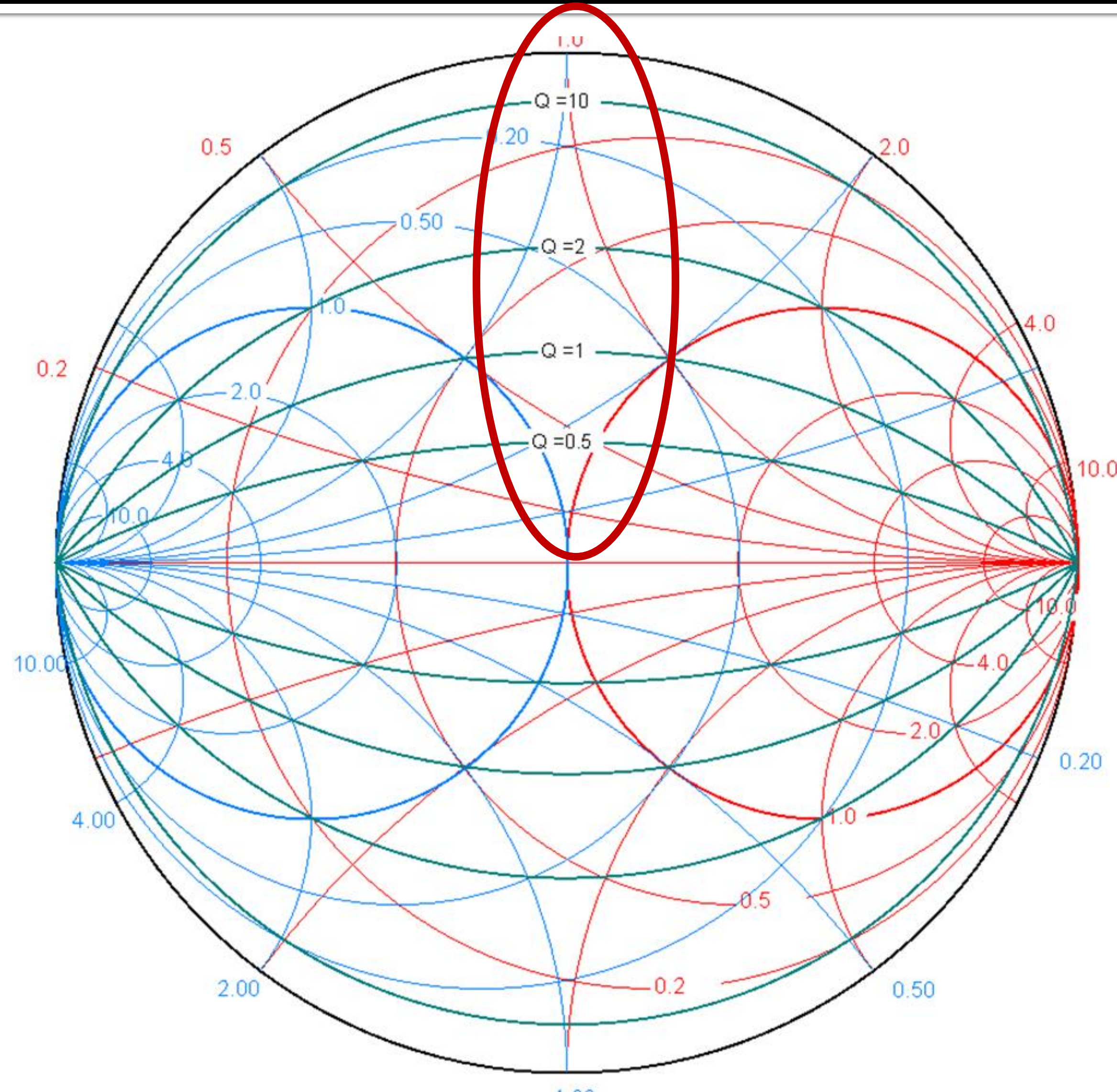


Constant Q circles

- Quality factor Q

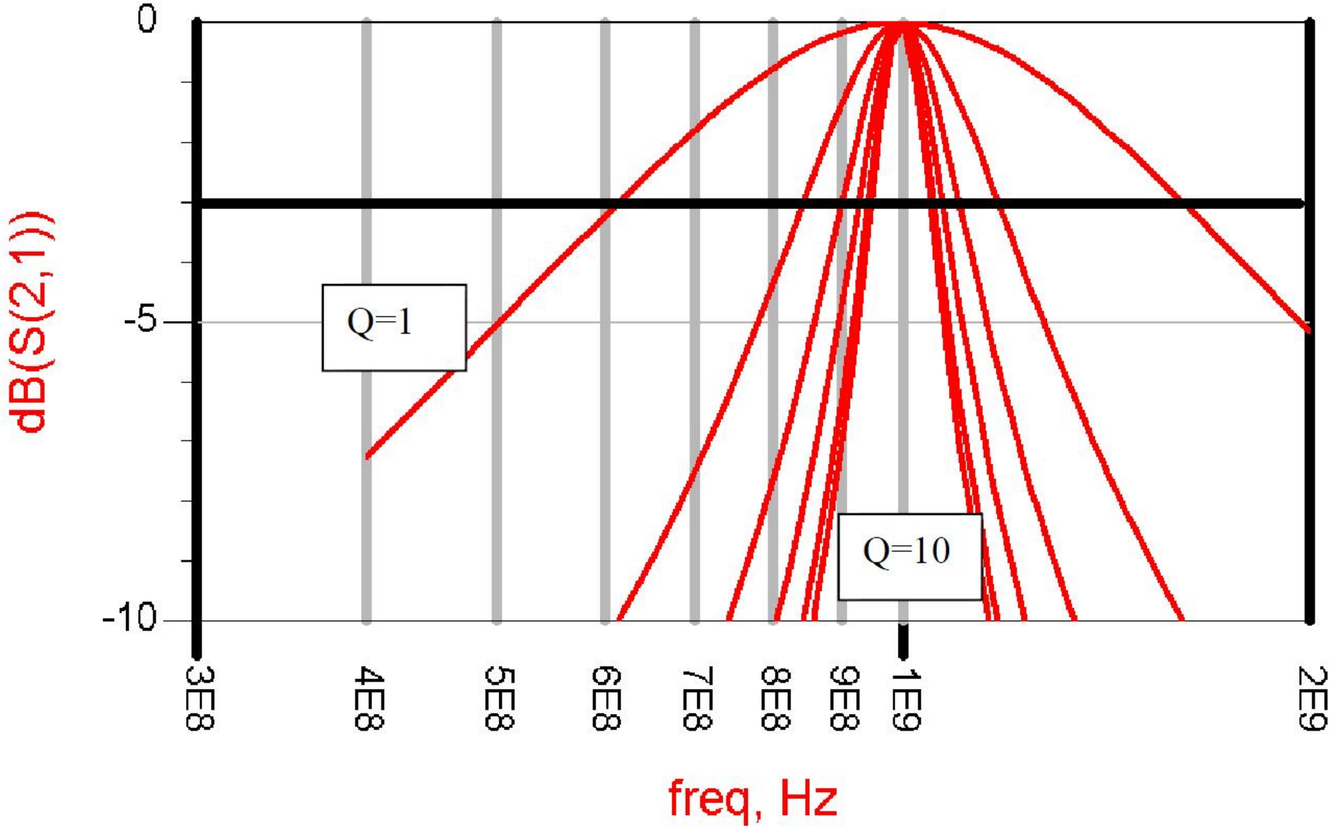
$$Q = \frac{X}{R} = \frac{G}{B} = const$$





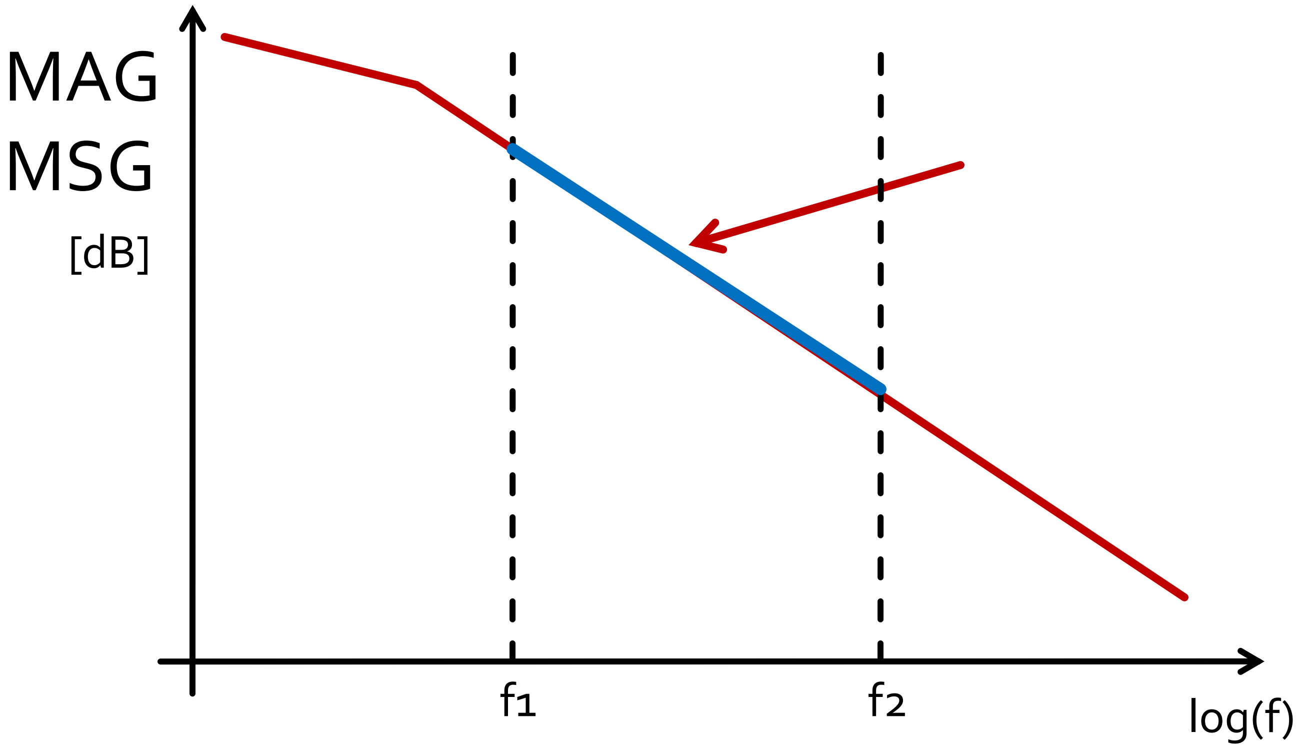
Quality factor - bandwidth

- High quality factor is equivalent with narrow bandwidth



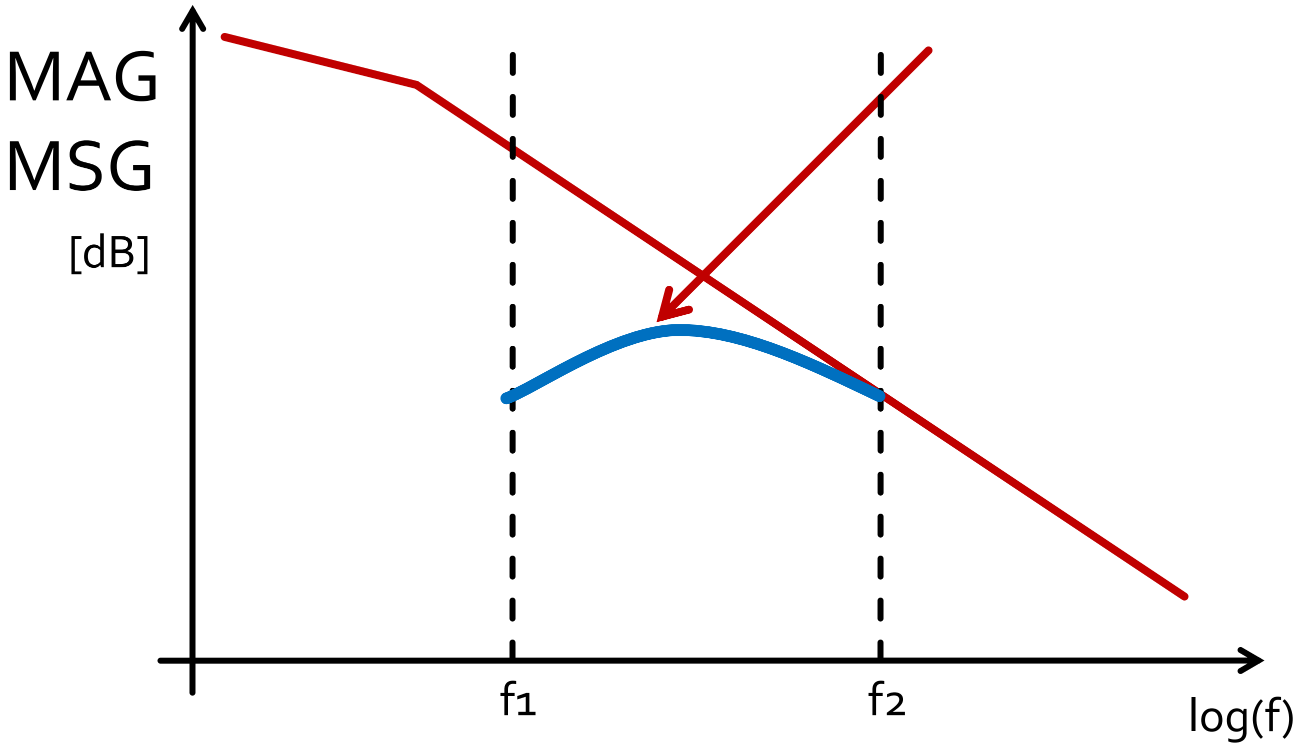
Wide bandwidth amplifier

- Design for maximum gain at two different frequencies creates an frequency unbalanced amplifier



Wide bandwidth amplifier

- Design for maximum gain at highest frequency
- Controlled mismatch at lower frequency
  - eventually at more frequencies inside the bandwidth



Design for Specified Gain

- Assumes the amplifier device **unilateral**

Input and output can be treated independently

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$\boxed{S_{12} \cong 0} \qquad \Gamma_{in} = S_{11}$$

- Maximum power gain

$$\Gamma_S = S_{11}^*$$

$$\Gamma_L = S_{22}^*$$

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2}$$

Unilateral figure of merit

- Allows estimation of the error introduced by the unilateral assumption

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} \qquad U = \frac{|S_{12}|\cdot|S_{21}|\cdot|S_{11}|\cdot|S_{22}|}{\left(1-|S_{11}|^2\right)\cdot\left(1-|S_{22}|^2\right)}$$

- We compute U then the maximum and minimum deviation of G_{TU} from G_T
  - this deviation must be accounted in the design as a reserve gain against the target gain

$$-20\cdot\log(1+U) < G_T[dB] - G_{TU}[dB] < -20\cdot\log(1-U)$$

Example

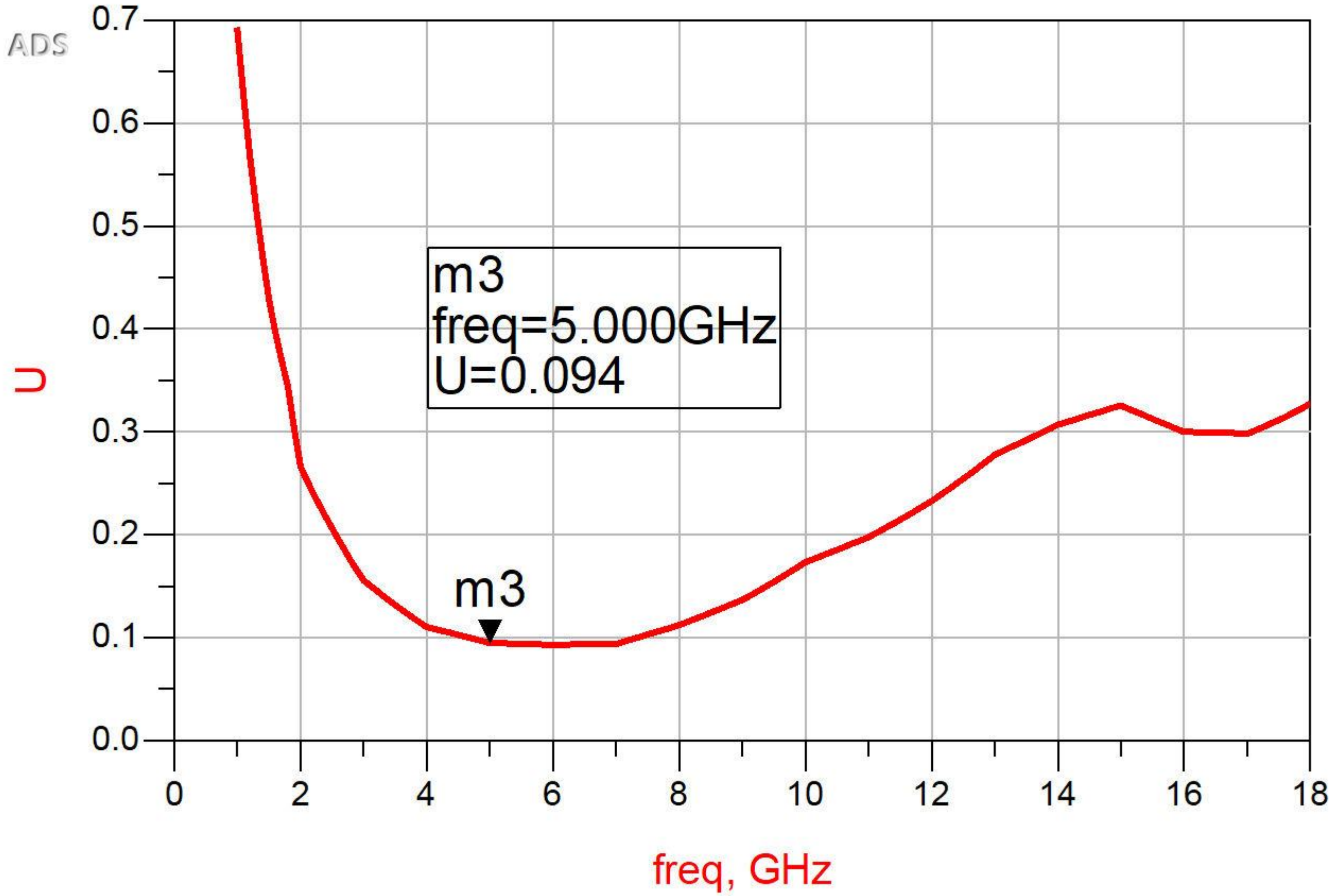
- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz
  - S11 = 0.64∠139°
  - S12 = 0.119∠-21°
  - S21 = 3.165 ∠16°
  - S22 = 0.22 ∠146°

$$U = \frac{|S_{12}|\cdot|S_{21}|\cdot|S_{11}|\cdot|S_{22}|}{\left(1-|S_{11}|^2\right)\cdot\left(1-|S_{22}|^2\right)} = 0.094$$

$$-0.783\, dB < G_T[dB] - G_{TU}[dB] < 0.861\, dB$$

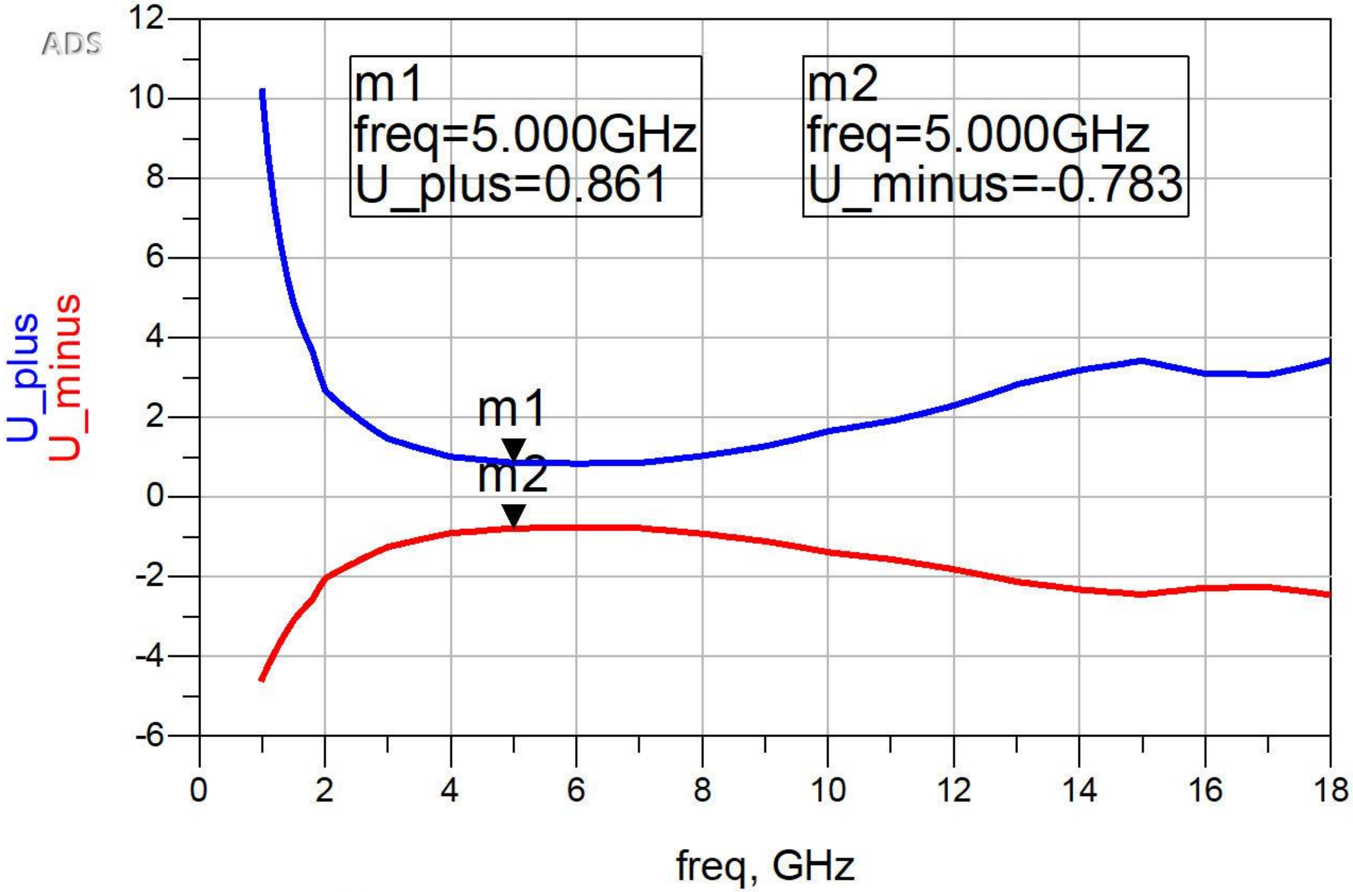
Example

- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz

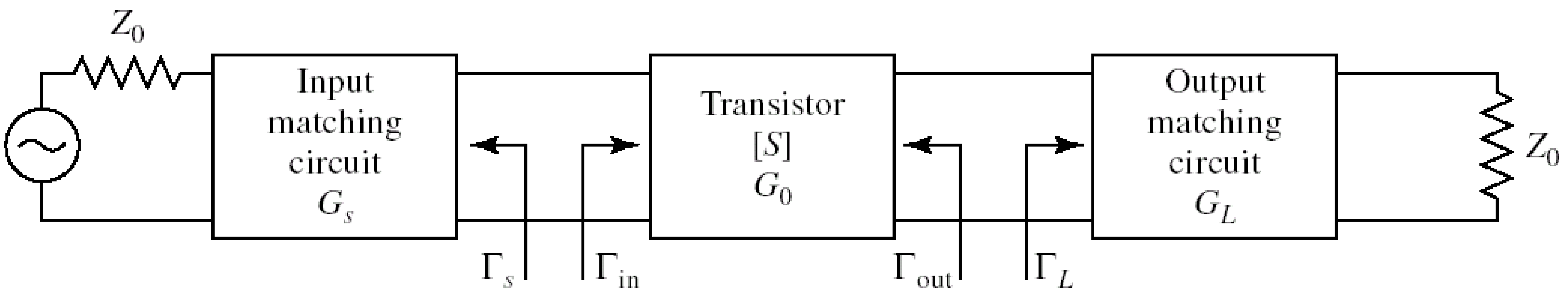


Example

- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz, maximum and minimum deviation [dB]



Design for Specified Gain



- In the unilateral assumption:

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}$$

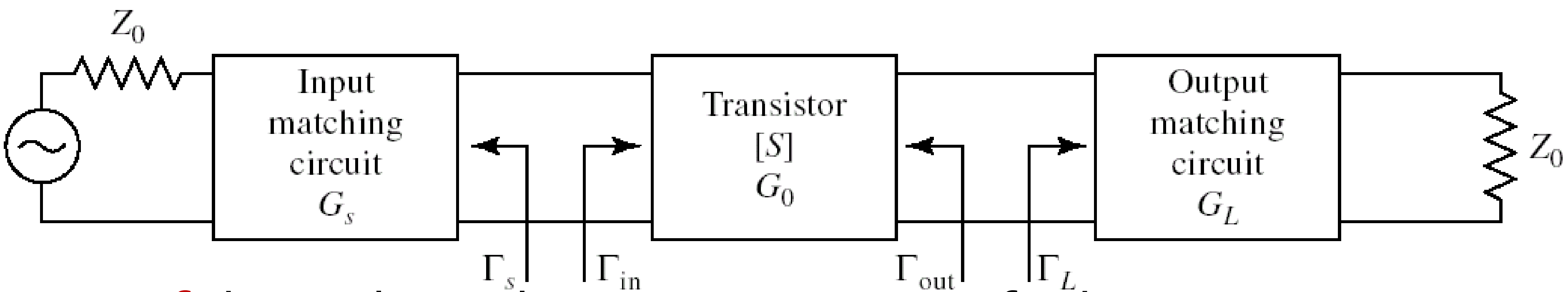
$$G_0 = |S_{21}|^2$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$G_S = G_S(\Gamma_S)$$

$$G_L = G_L(\Gamma_L)$$

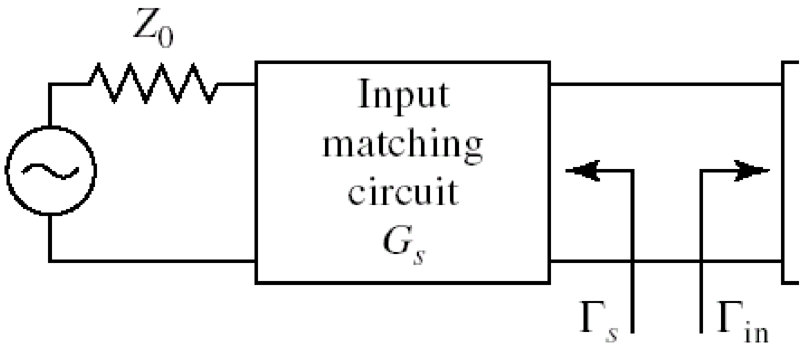
Design for Specified Gain



- **If** the unilateral assumption is justified :
  - power gain added by the input matching circuit **is not** influenced by the output matching circuit $G_S = G_S(\Gamma_S)$
  - power gain added by the output matching circuit **is not** influenced by the input matching circuit $G_L = G_L(\Gamma_L)$
- Output /Input match can be designed **independently**
  - We can impose different demands for input/output
  - Total gain is:

$$G_T = G_S \cdot G_0 \cdot G_L \qquad G_T[dB] = G_S[dB] + G_0[dB] + G_L[dB]$$

Input matching circuit



$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}$$

- Maximum gain in the case of complex conjugate match

$$\Gamma_S = S_{11}^* \Rightarrow \qquad G_{S\max} = \frac{1}{1-|S_{11}|^2}$$

- For any other input matching circuit:

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} < G_{S\max} = \frac{1}{1-|S_{11}|^2}$$

Example

- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz
  - S11 = 0.64∠139°
  - S12 = 0.119∠-21°
  - S21 = 3.165 ∠16°
  - S22 = 0.22 ∠146°

$$U = \frac{|S_{12}|\cdot|S_{21}|\cdot|S_{11}|\cdot|S_{22}|}{\left(1-|S_{11}|^2\right)\cdot\left(1-|S_{22}|^2\right)} = 0.094$$

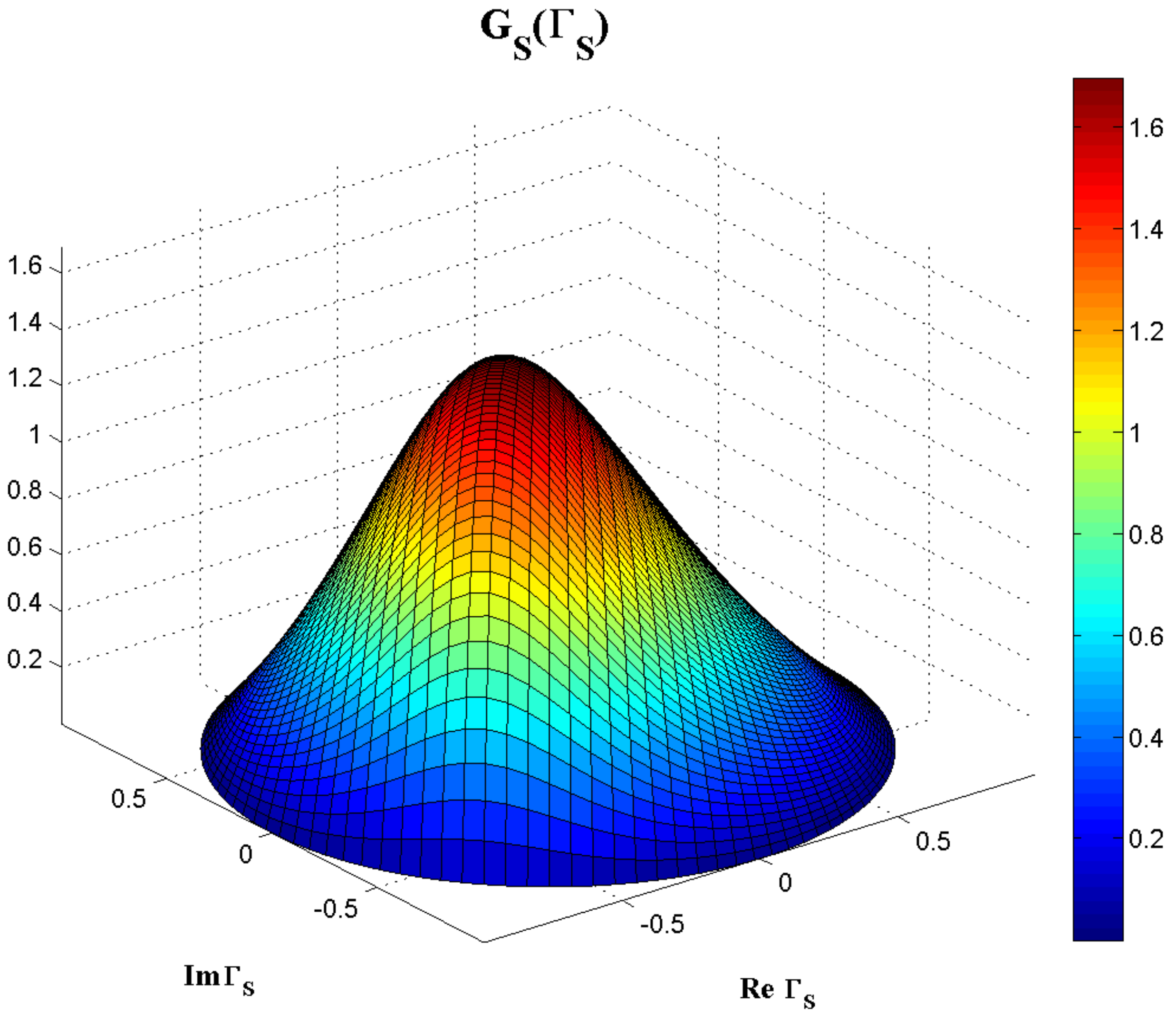
$$-0.783\,dB < G_T[dB] - G_{TU}[dB] < 0.861\,dB$$

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2}\cdot|S_{21}|^2\cdot\frac{1}{1-|S_{22}|^2} = 17.83$$

$$G_{TU\max}[dB] = 12.511\,dB$$

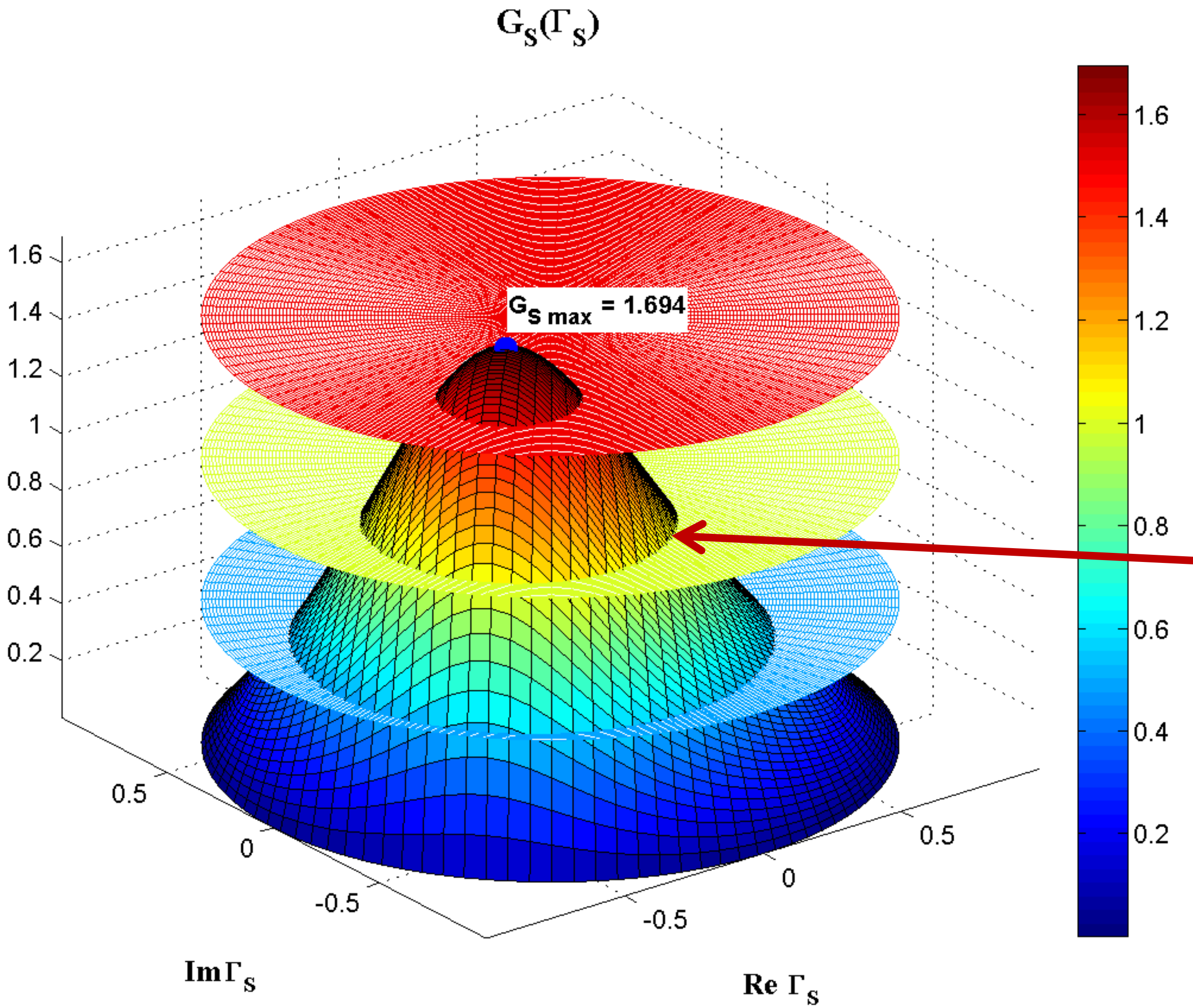
$$G_{S\max} = \frac{1}{1-|S_{11}|^2} = 1.694 = 2.289\,dB$$

$G_S(\Gamma_S)$



$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2}$$

$G_S(\Gamma_S)$, constant value contours



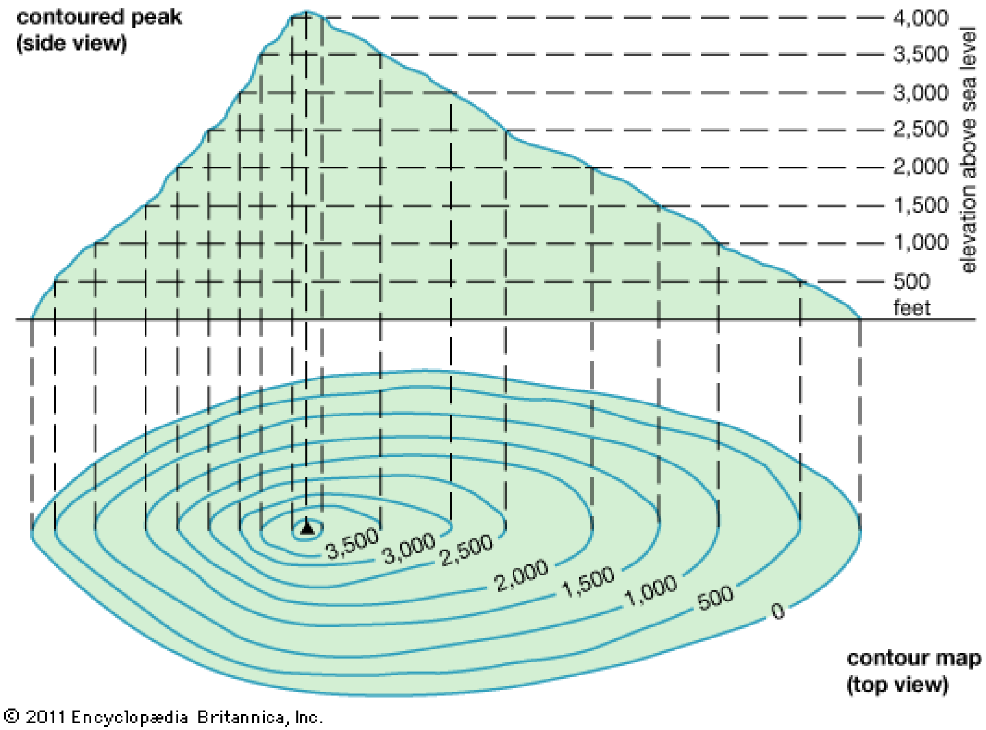
$G_S = 1.5$

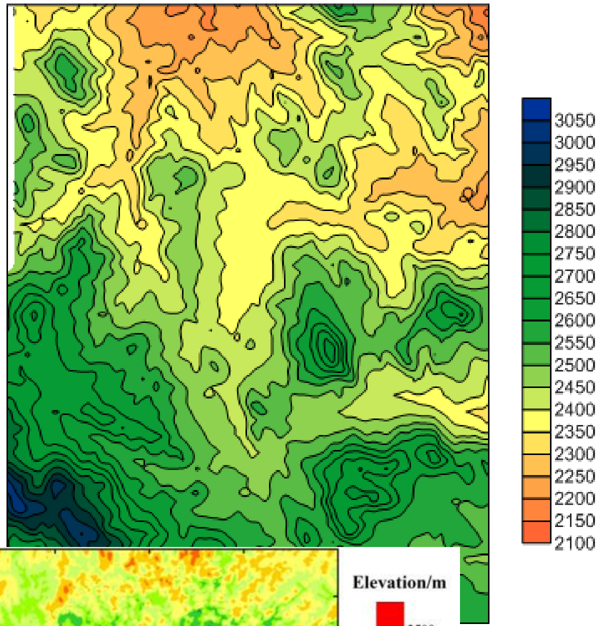
$G_S = 1.0$

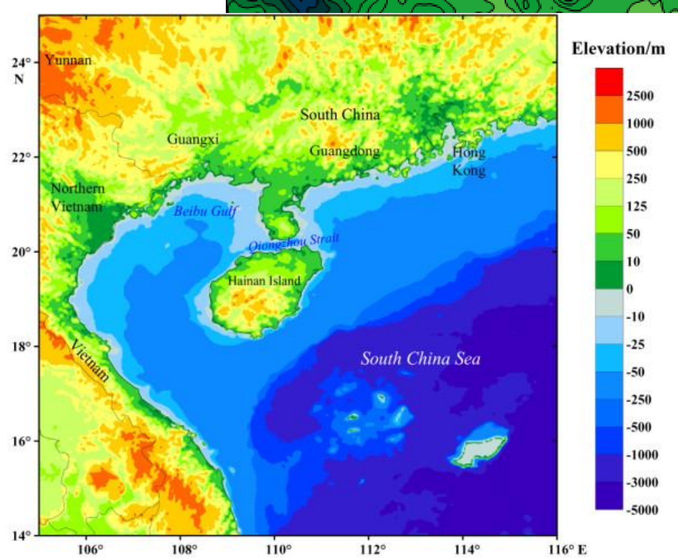
$G_S = 0.5$

Circles

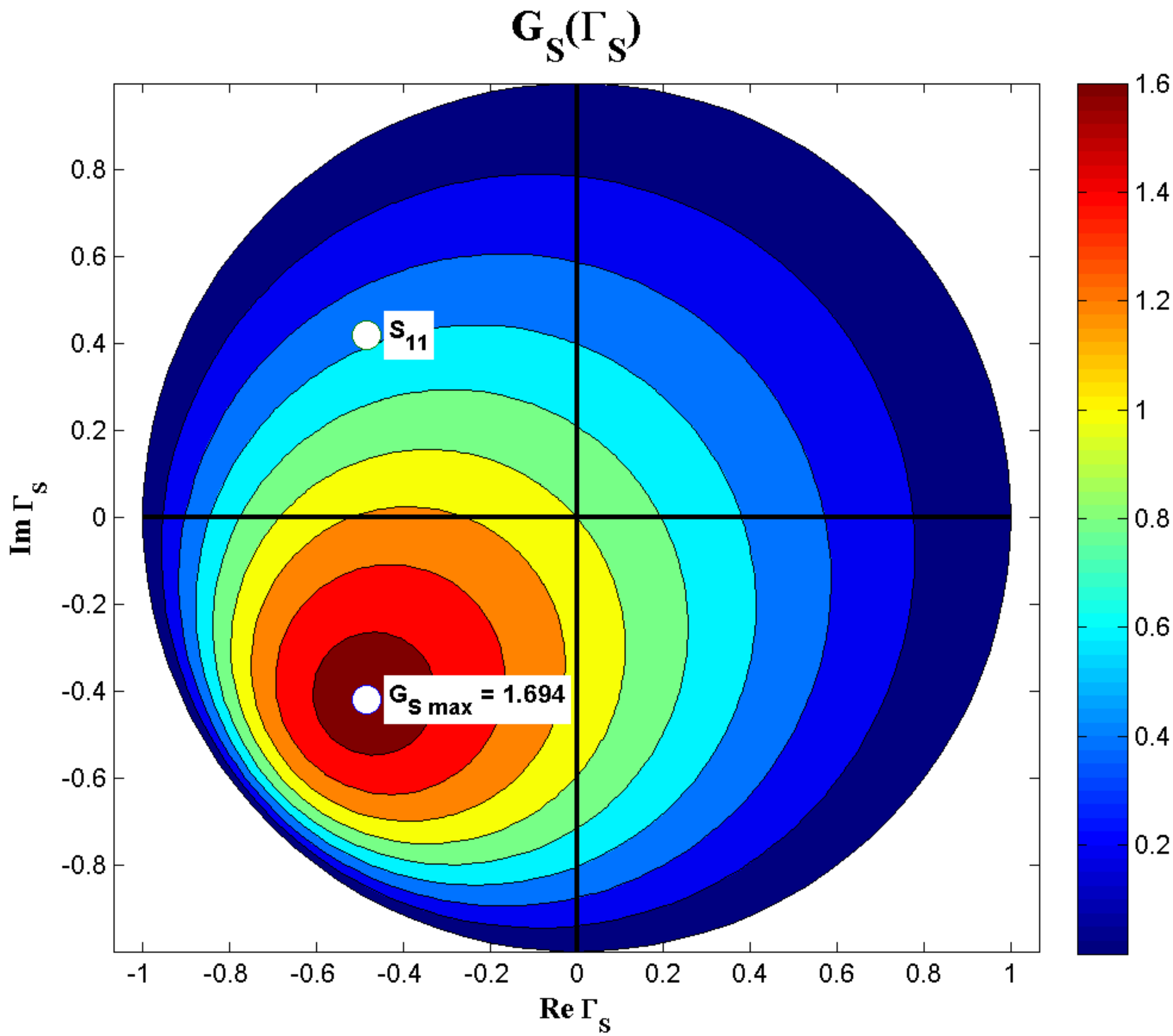
Contour map/lines







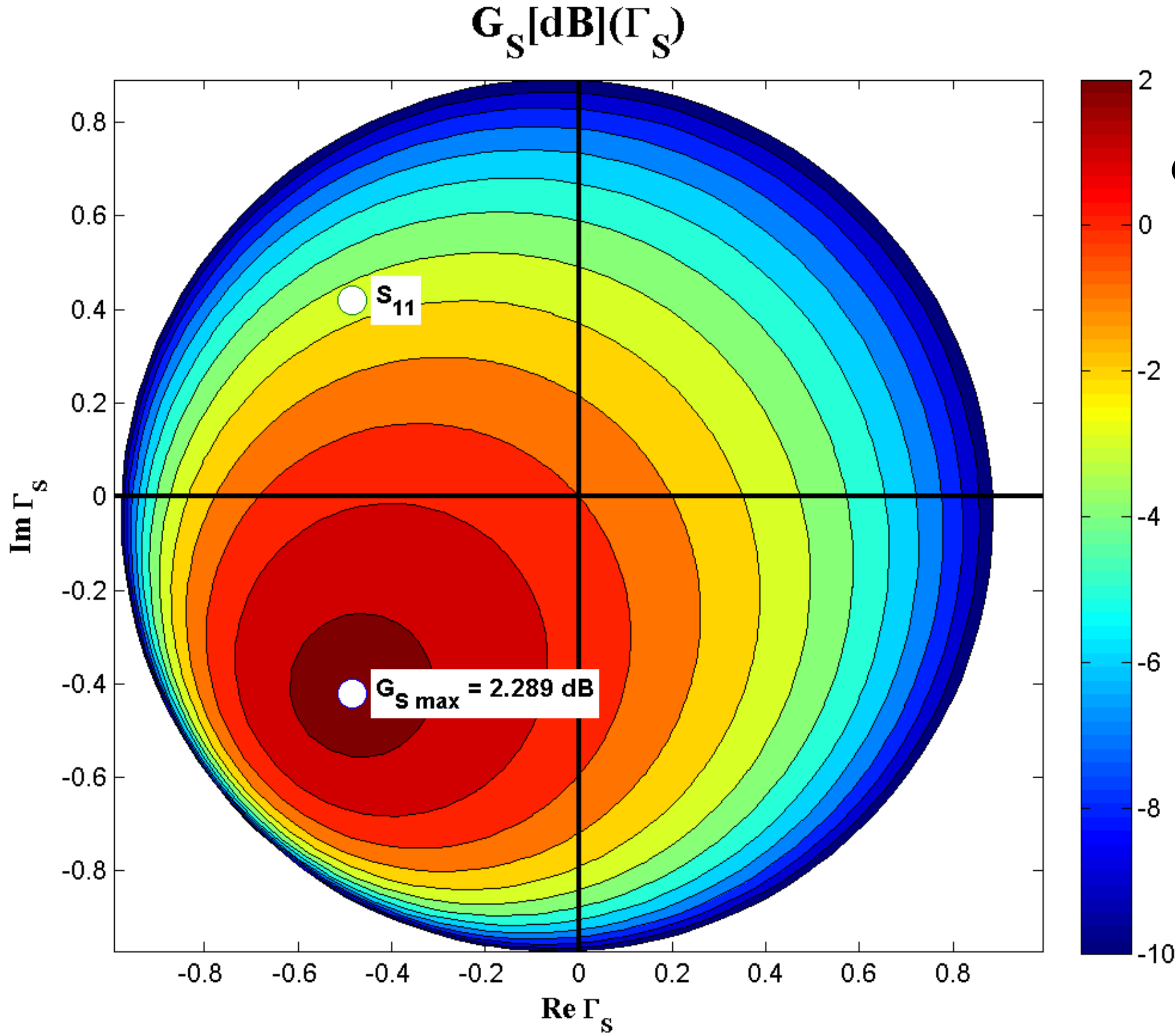
$G_S(\Gamma_S)$, constant value contours



$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}$$

$$G_{S\max} = G_S\big|_{\Gamma_S = S_{11}^*}$$

$G_S[dB](\Gamma_S)$, constant value contours



$$G_S[dB] = 10 \cdot \log\left(\frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\right)$$

$$G_{S\max} = G_S\big|_{\Gamma_S = S_{11}^*}$$

Input section constant gain circles

- The normalized gain factor (**linear scale!**)

$$g_S = \frac{G_S}{G_{S\max}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\cdot\left(1-|S_{11}|^2\right) < 1$$

- Locus of the points with fixed values $g_s<1$

$$g_S \cdot |1-S_{11}\cdot\Gamma_S|^2 = \left(1-|\Gamma_S|^2\right)\cdot\left(1-|S_{11}|^2\right)$$

$$\left(g_S\cdot|S_{11}|^2 + 1 - |S_{11}|^2\right)\cdot|\Gamma_S|^2 - g_S\cdot\left(S_{11}\cdot\Gamma_S + S_{11}^*\cdot\Gamma_S^*\right) = 1-|S_{11}|^2 - g_S$$

$$\Gamma_S\cdot\Gamma_S^* - \frac{g_S\cdot\left(S_{11}\cdot\Gamma_S + S_{11}^*\cdot\Gamma_S^*\right)}{1-(1-g_S)\cdot|S_{11}|^2} = \frac{1-|S_{11}|^2-g_S}{1-(1-g_S)\cdot|S_{11}|^2} \Bigg/ + \frac{g_S^2\cdot|S_{11}|^2}{\left[1-(1-g_S)\cdot|S_{11}|^2\right]^2}$$

$$|a+b|^2 = (a+b)\cdot(a+b)^* = (a+b)\cdot\left(a^*+b^*\right) = \underline{|a|^2} + |b|^2 + \underline{a^*\cdot b} + \underline{a\cdot b^*}$$

Plan complex – geometrie analitica

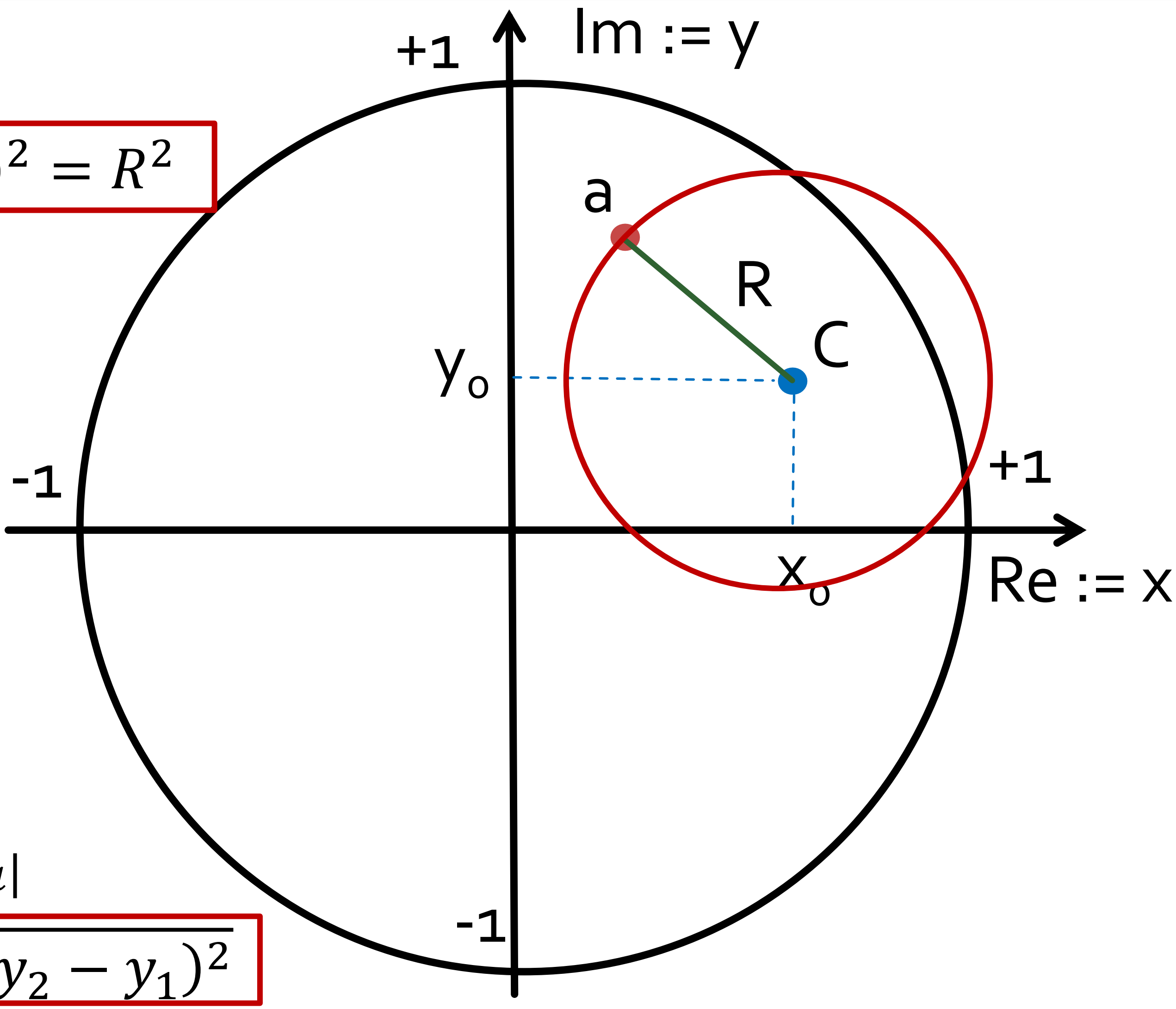
- solutii

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$a = x + j \cdot y$$

$$C = x_0 + j \cdot y_0$$

$$D(a, C) = |a - C| = R$$



$$a = x_1 + j \cdot y_1$$

$$b = x_2 + j \cdot y_2$$

$$D(a, b) = |a - b| = |b - a|$$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Input section constant gain circles

$$\left|\Gamma_S - \frac{g_S \cdot S_{11}^*}{1-(1-g_S)\cdot|S_{11}|^2}\right| = \frac{\sqrt{1-g_S}\cdot\left(1-|S_{11}|^2\right)}{1-(1-g_S)\cdot|S_{11}|^2} \qquad |\Gamma_S - C_S| = R_S$$

$$C_S = \frac{g_S \cdot S_{11}^*}{1-(1-g_S)\cdot|S_{11}|^2} \qquad R_S = \frac{\sqrt{1-g_S}\cdot\left(1-|S_{11}|^2\right)}{1-(1-g_S)\cdot|S_{11}|^2}$$

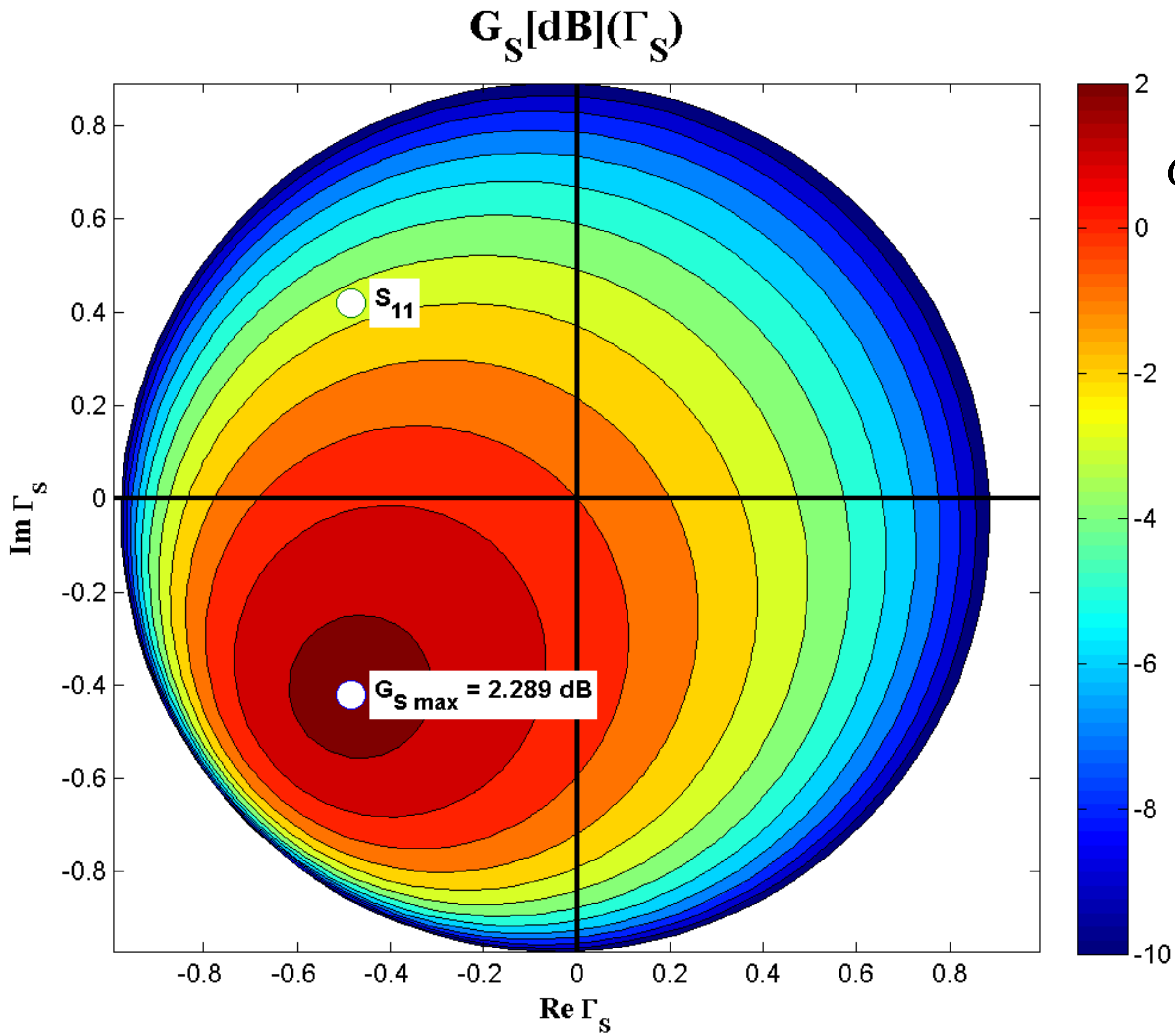
- Equation of a circle in the complex plane where Γ_S is plotted
- **Interpretation**: Any reflection coefficient Γ_S which plotted in the complex plane lies **on** the circle drawn for $g_{circle} = G_{circle}/G_{Smax}$ will lead to a gain $G_S = G_{circle}$
  - Any reflection coefficient Γ_S plotted **outside** this circle will lead to a gain $G_S < G_{circle}$
  - Any reflection coefficient Γ_S plotted **inside** this circle will lead to a gain $G_S > G_{circle}$

Input section constant gain circles

$$C_S = \frac{g_S \cdot S_{11}^*}{1-(1-g_S)\cdot |S_{11}|^2} \qquad R_S = \frac{\sqrt{1-g_S}\cdot\left(1-|S_{11}|^2\right)}{1-(1-g_S)\cdot |S_{11}|^2}$$

- The centers of each family of circles lie along straight lines given by the angle of $\Gamma_{S\max} = S_{11}^*$
- Circles are plotted (traditionally, CAD) in **logarithmic scale** ([dB])
  - formulas are in **linear scale!**
- The circle for G_S = 0 dB will always pass through the origin of the complex plane (center of the Smith chart)

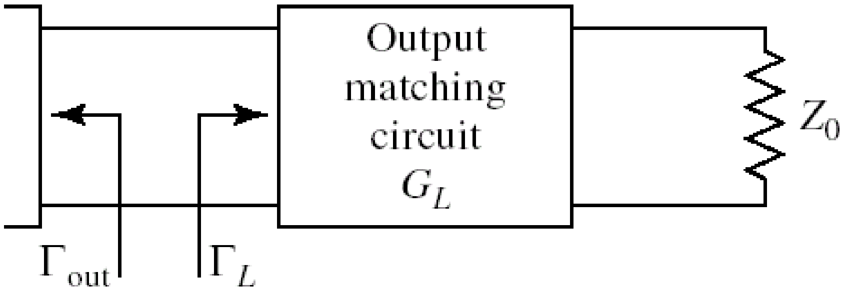
$G_S[dB](\Gamma_S)$, constant value contours



$$G_S[dB] = 10 \cdot \log\left(\frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\right)$$

$$G_{S\max} = G_S\Big|_{\Gamma_S = S_{11}^*}$$

Output section constant gain circles



$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

- Maximum gain for $\Gamma_L = S_{22}^* \Rightarrow$ $G_{L\max} = \dfrac{1}{1-|S_{22}|^2}$

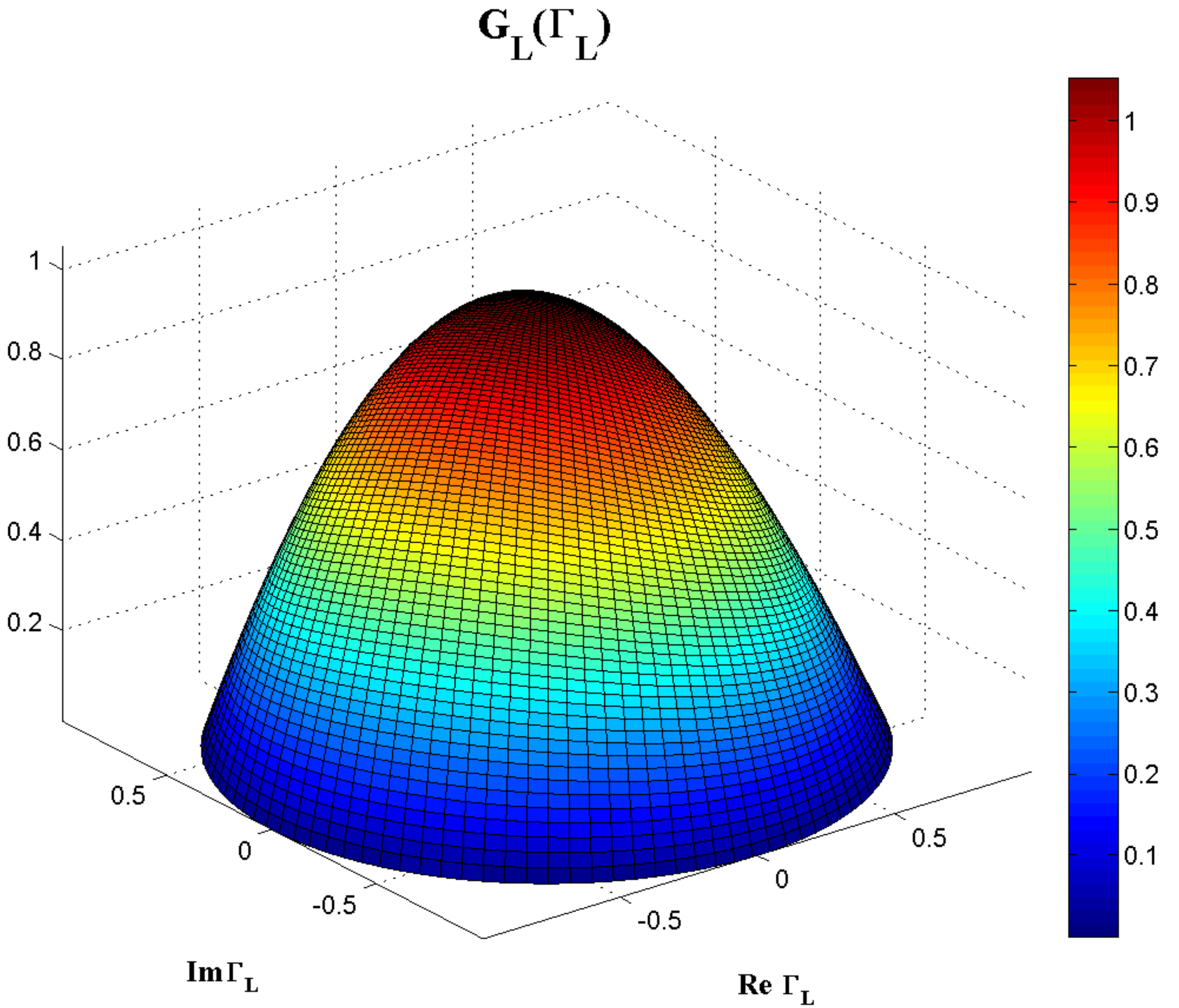
$$g_L = \frac{G_L}{G_{L\max}} = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}\cdot\left(1-|S_{22}|^2\right) < 1$$

- Similar computations

$$C_L = \frac{g_L\cdot S_{22}^*}{1-(1-g_L)\cdot|S_{22}|^2} \qquad R_L = \frac{\sqrt{1-g_L}\cdot\left(1-|S_{22}|^2\right)}{1-(1-g_L)\cdot|S_{22}|^2}$$

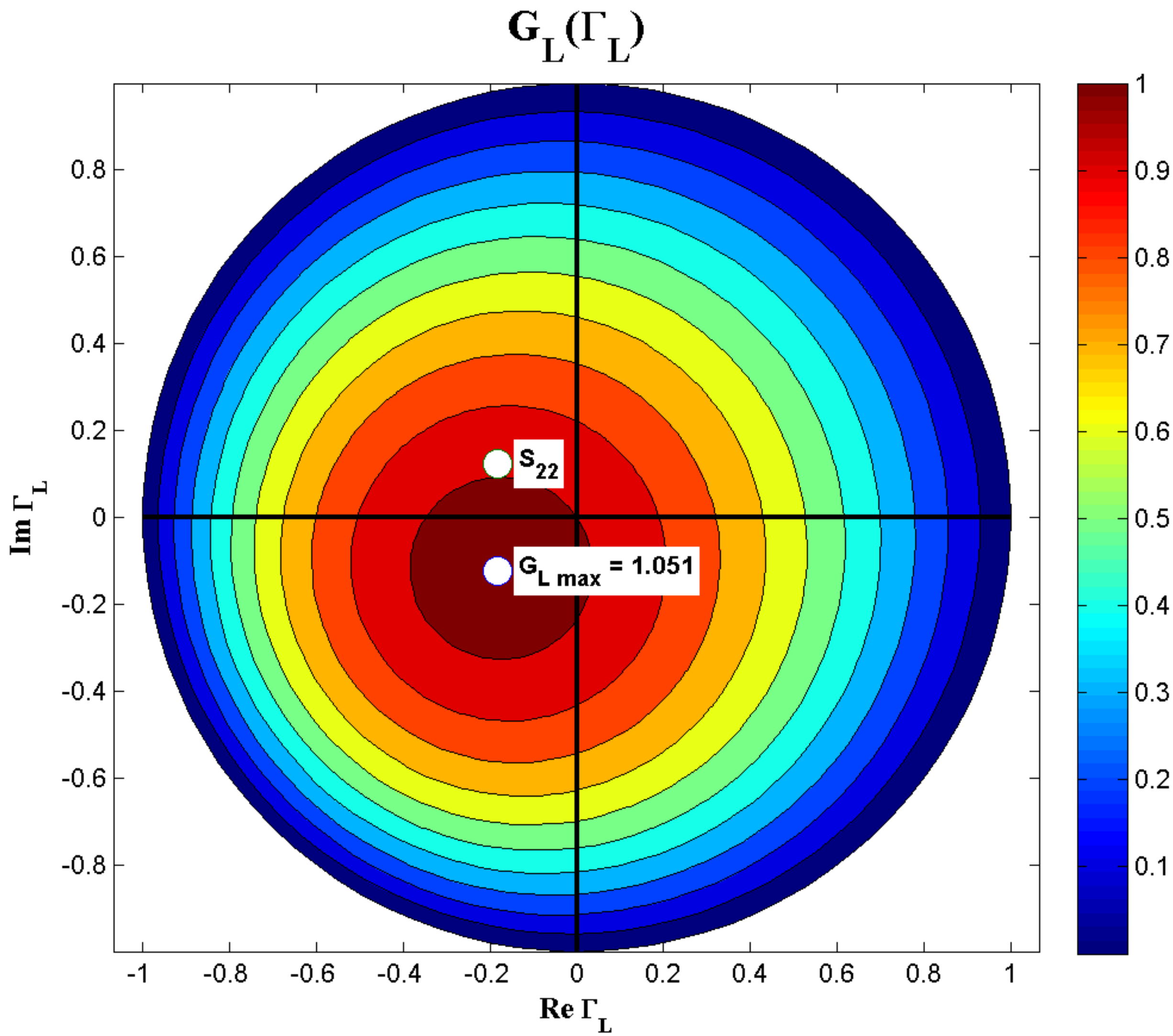
- Example $G_{L\max} = \dfrac{1}{1-|S_{22}|^2} = 1.051 = 0.215\ dB$

$G_L(\Gamma_L)$



$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

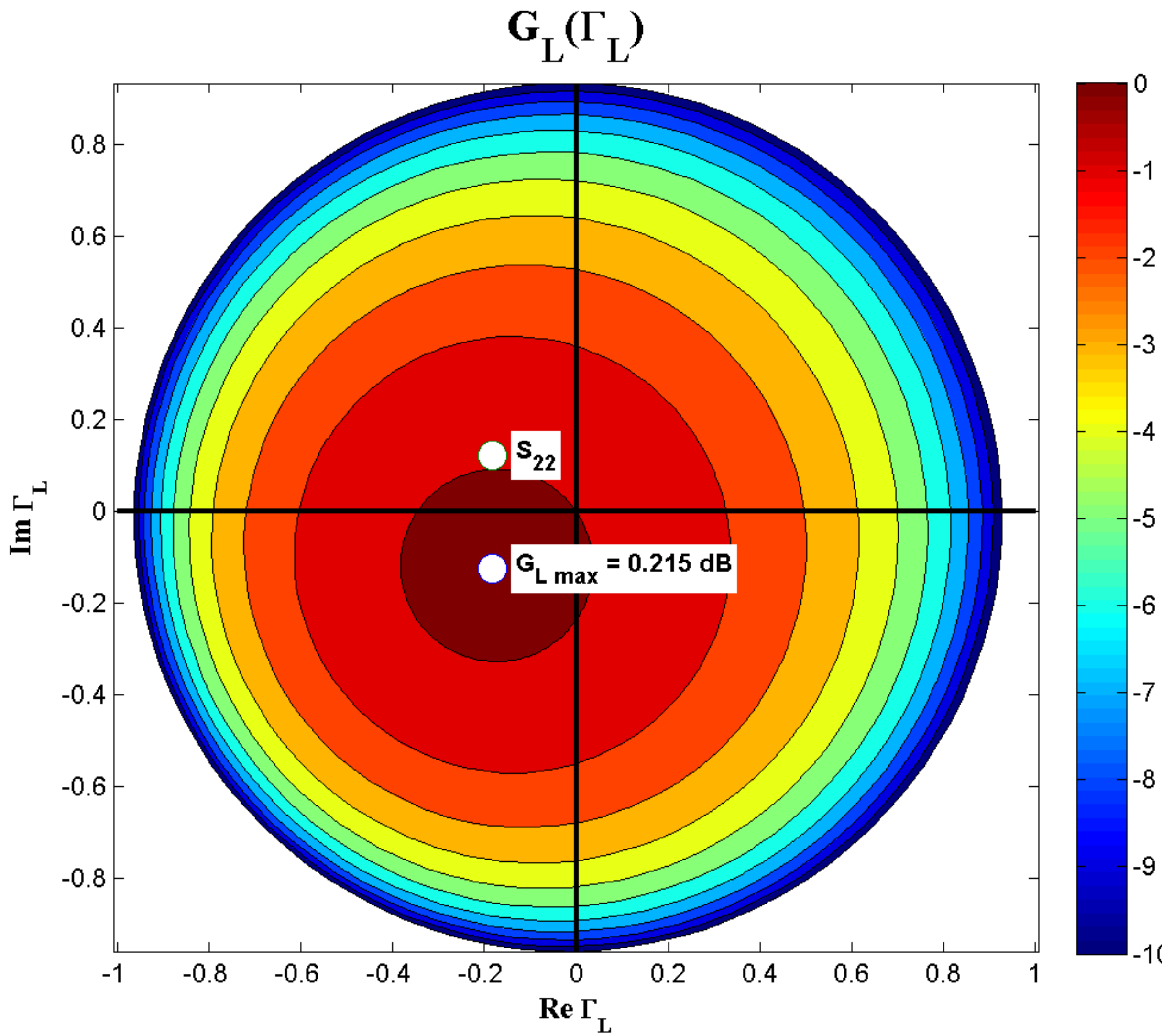
$G_L(\Gamma_L)$, constant value contours



$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$G_{L\max} = G_L\Big|_{\Gamma_L=S_{22}^*}$$

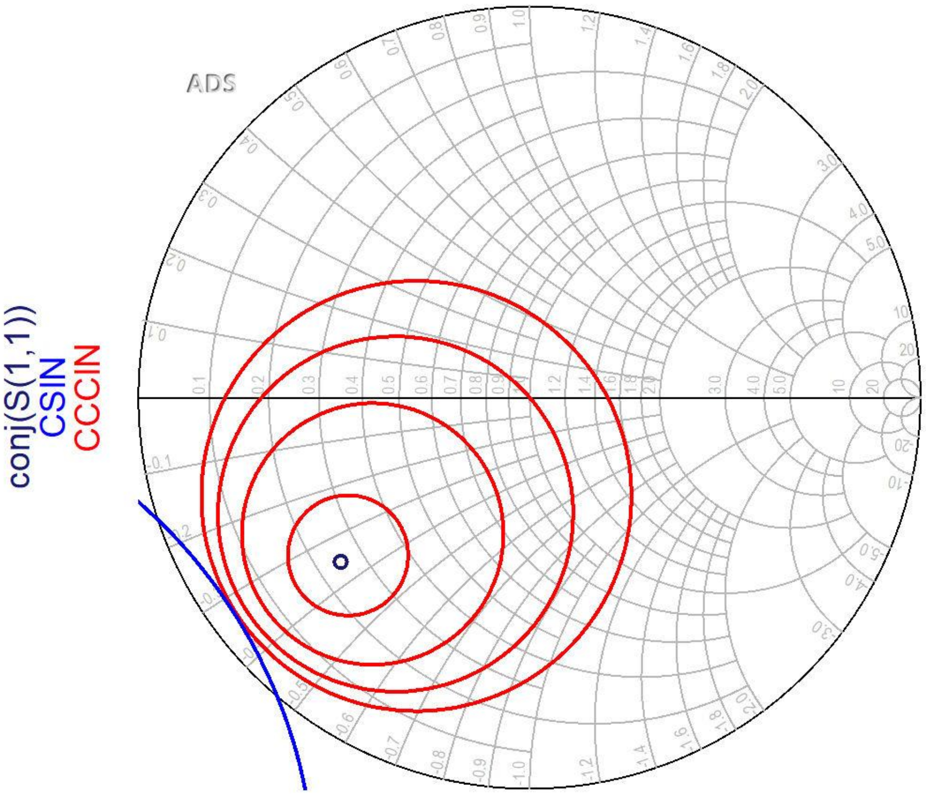
$G_L[dB](\Gamma_L)$, constant value contours

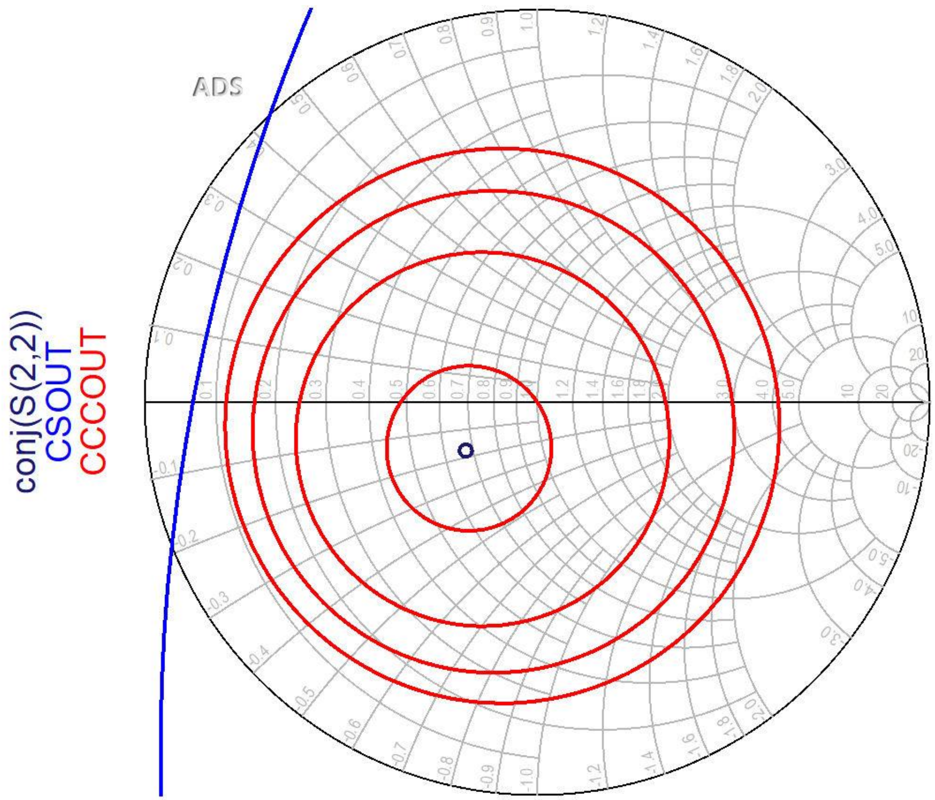


$$G_L[dB] = 10 \cdot \log\left(\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}\right)$$

$$G_{L\max} = G_L\Big|_{\Gamma_L = S_{22}^*}$$

ADS

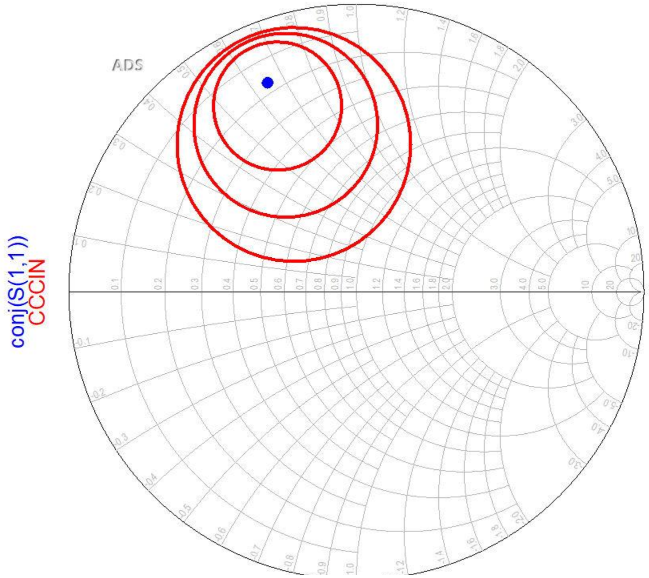


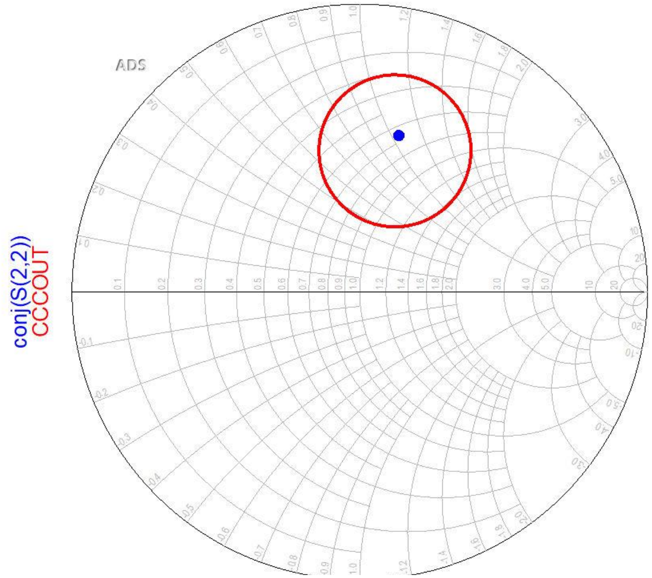


- Circles are plotted for requested values (**in dB!**)
- It is usefull to compute G_{Smax} and G_{Lmax} before
  - in order to request relevant circles

Schematic 1 – Lab 3

- **multiple circles (families)** are plotted and some required values are computed





Eqn gamma_opt=Sopt

Eqn G0=10*log(mag(S(2,1))**2)

Eqn GSmax=10*log(1/(1-mag(S(1,1))**2))

Eqn GLmax=10*log(1/(1-mag(S(2,2))**2))

freq	K	MAG	NFmin	Sopt	Rn	G0	GLmax	GSmax
5.000 GHz	0.533	15.296	0.700	0.660 / 106....	19.500	8.974	1.634	4.249

Design for Specified Gain

- We compute G_o, G_{Smax} , G_{Lmax}
- To obtain the design gain we **choose** supplemental gain needed (supplemental to constant G_o)
  - we account for the deviation that might arise from the unilateral assumption (using unilateral figure of merit U)

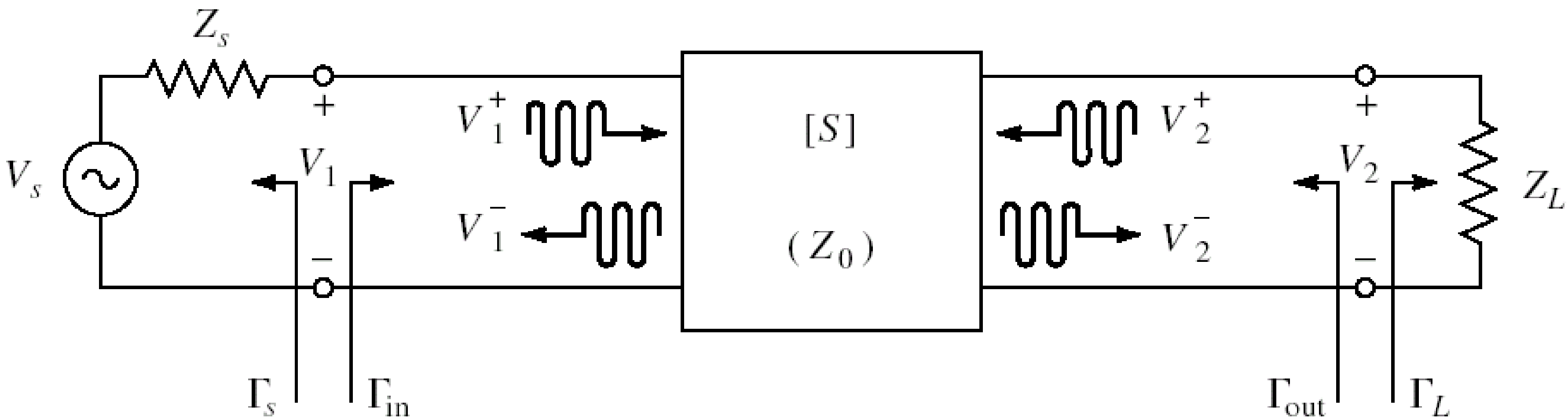
$$G_{design}[dB] = G_{S\_design}[dB] + G_0[dB] + G_{L\_design}[dB]$$

- We plot the circles for design (chosen) values G_{S_design} , G_{L_design}
- We design input and output matching circuits which move the reflection coefficient **on** or **inside** the design circles (depending on specific application requirements)

Microwave Amplifiers

Low-Noise Amplifier Design

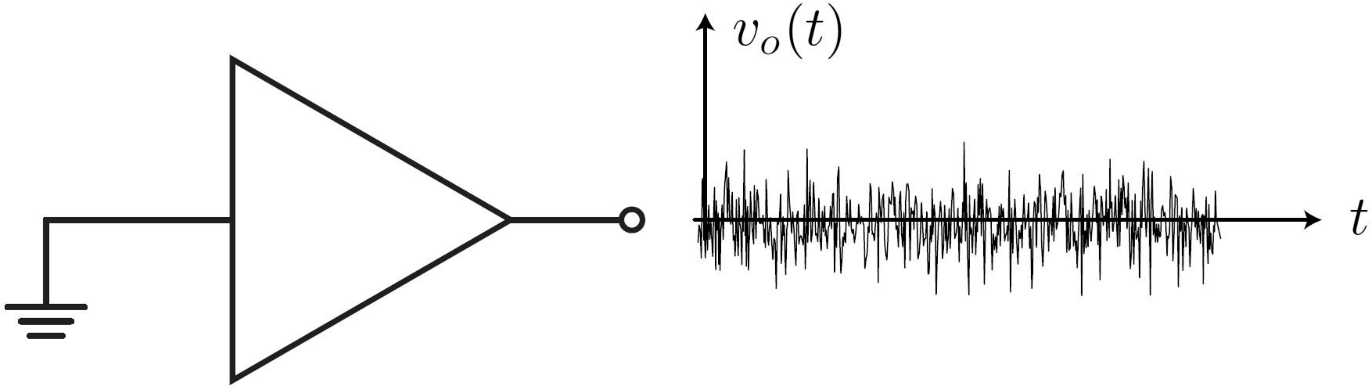
Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - power gain
  - **noise** (sometimes – **small signals**)
  - linearity (sometimes – large signals)

Noise

- Noise: random fluctuations of the signal

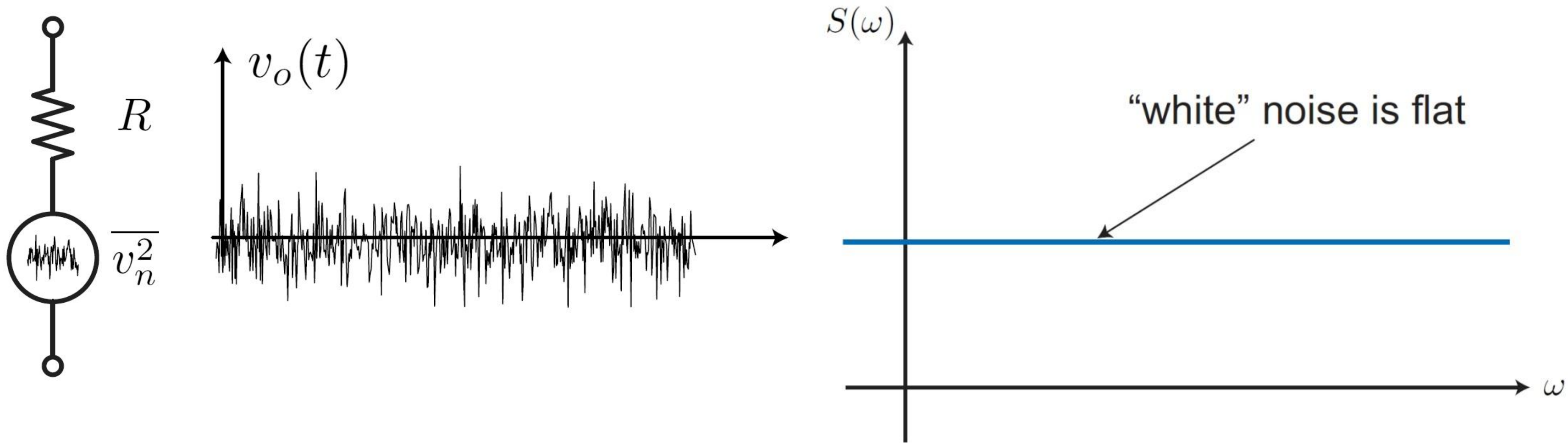


$$\overline{v_n(t)} = \langle v_n(t) \rangle = \frac{1}{T}\int_0^T v_n(t)\,dt = 0$$

$$\overline{v_n^2(t)} = \langle v_n^2(t) \rangle = \frac{1}{T}\int_0^T v_n^2(t)\,dt \neq 0$$

$$V_{n(ef)} = \sqrt{\overline{v_n^2(t)}}$$

Noise



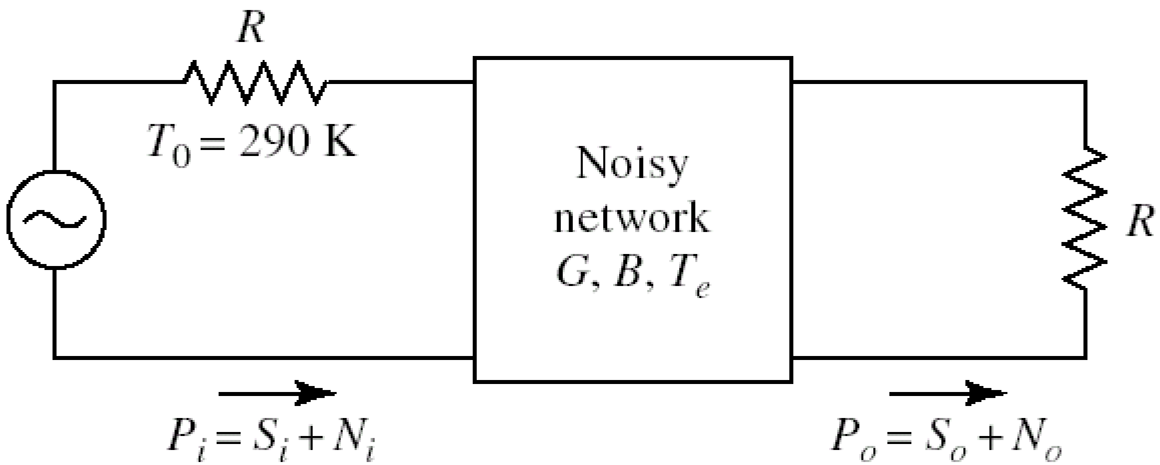
- effective noise voltage

$$V_{n(ef)} = \sqrt{4kTBR}$$

- noise power available (for maximum power transfer with impedance/resistance matching)

$$P_n = kTB$$

Noise Figure F



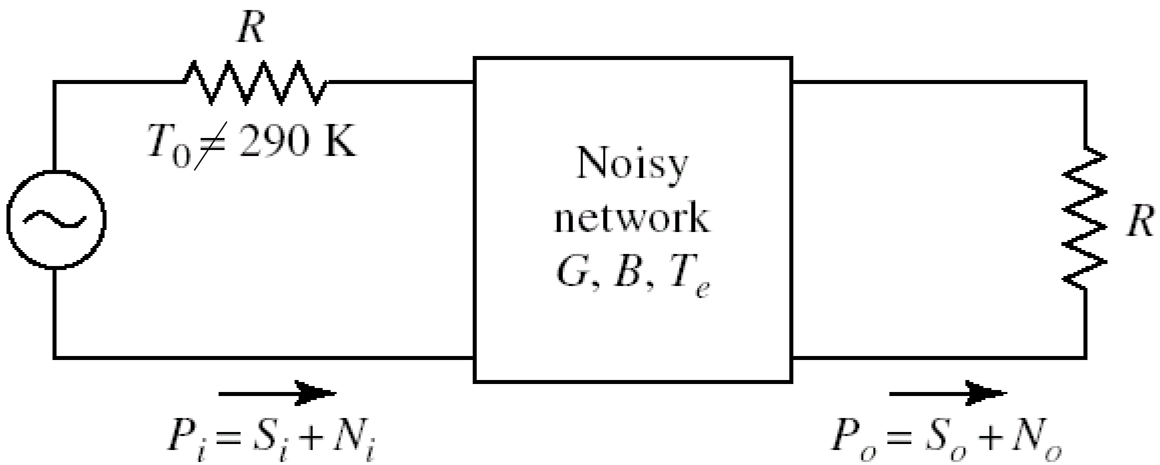
- The noise figure F, is a measure of the reduction in signal-to-noise ratio between the input and output of a device, when (by definition) the input noise power is assumed to be the noise power resulting from a matched resistor at To = 290 K (reference noise conditions)

$$F = \left.\frac{S_i / N_i}{S_o / N_o}\right|_{T_0 = 290K}$$

$$V_{n(ef)} = \sqrt{4kTBR}$$

$$P_n = kTB$$

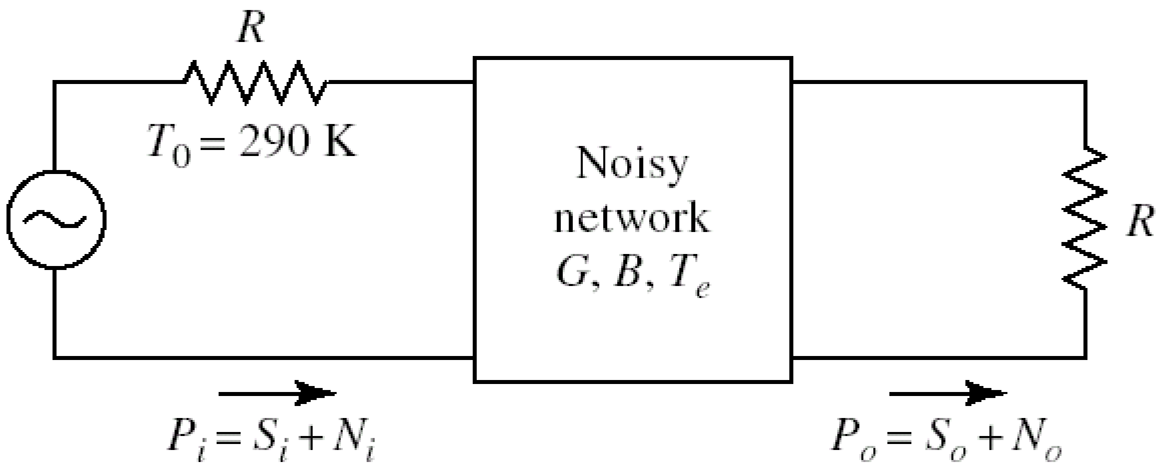
Noise Figure F



- The noise figure F, **is not** directly a measure of the reduction in signal-to-noise ratio between the input and output of a device, when the input noise power is different from that of the reference noise conditions

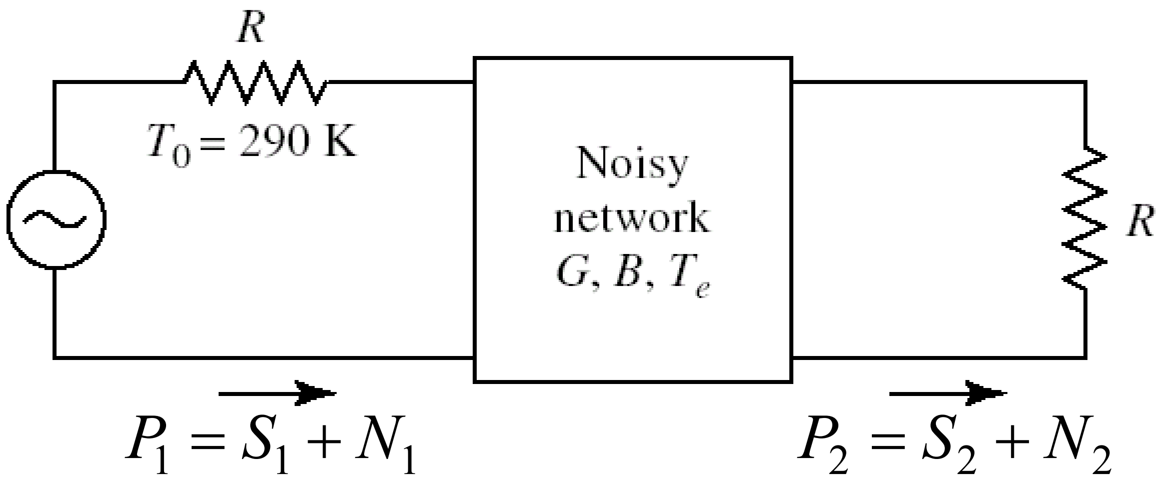
$$F \neq \left. \frac{S_i / N_i}{S_o / N_o} \right|_{T_0 \neq 290K}$$

Noise Figure F



- In general, the output noise power consists of two elements:
  - the input noise power amplified or attenuated by the device (for example amplified with the power gain G applied also to the desired signal)
  - a noise power generated internally by the network if the network is noisy (this power **does not** depend on the input noise power)

Noise Figure F



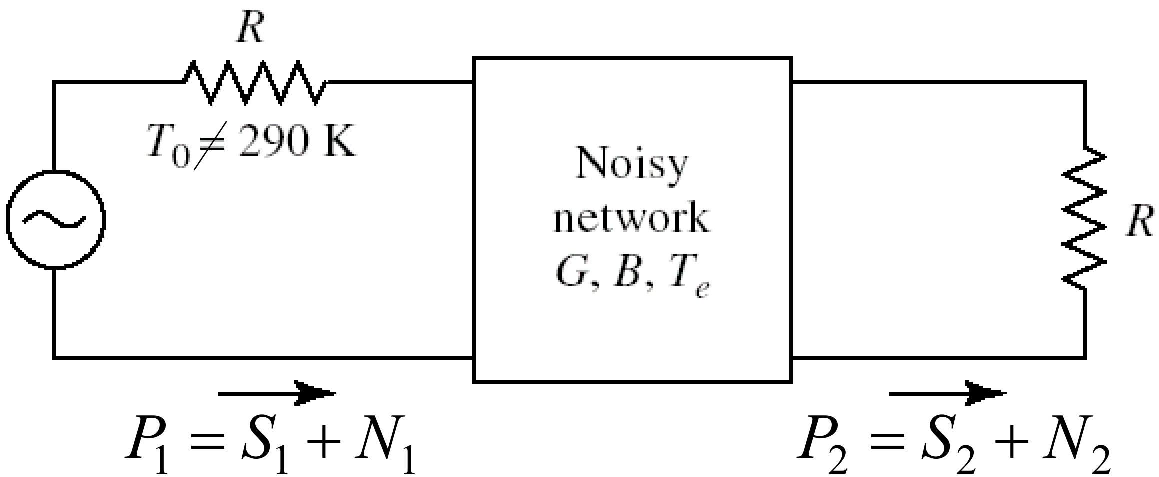
- Estimation of the internally generated noise power can be done using the Noise Figure F definition:

$$F = \left. \frac{S_1 / N_1}{S_2 / N_2} \right|_{T_0 = 290K, N_1 = N_0}$$

$$N_2 = F \cdot N_0 \cdot \frac{S_2}{S_1} = F \cdot N_0 \cdot G$$

$$N_2 = N_0 \cdot G + (F - 1) \cdot N_0 \cdot G$$

Noise Figure F

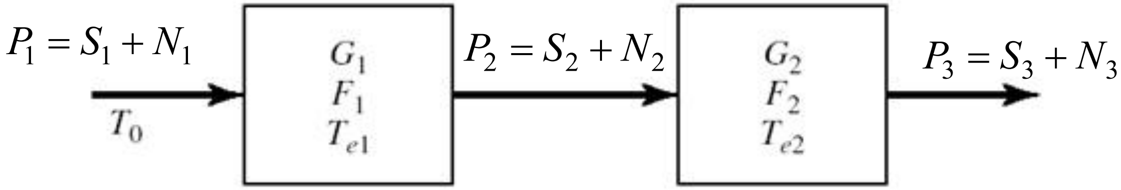


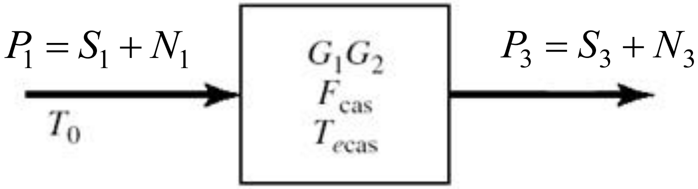
- We identify the two terms:
  - amplified input noise
  - internally generated noise
- When the input noise does not correspond to reference noise conditions (N1 ≠ No)
  - the internally generated noise does not change

$$N_2 = N_0 \cdot G + (F-1) \cdot N_0 \cdot G$$

$$N_2 = N_1 \cdot G + (F-1) \cdot N_0 \cdot G$$

Noise figure of a cascaded system





$$N_2 = N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1$$

$$N_3 = N_2 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

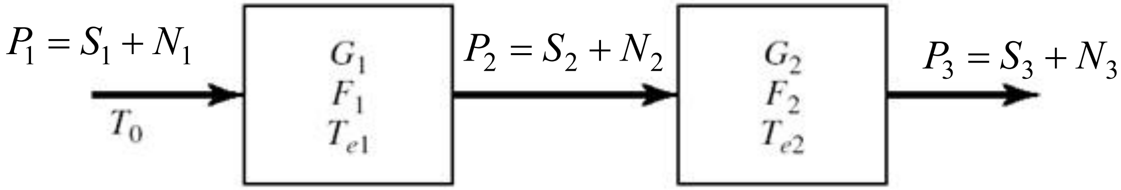
$$G_{cas} = G_1 \cdot G_2$$

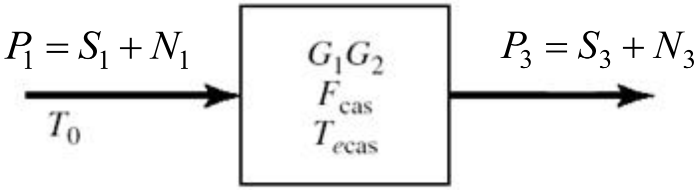
$$N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$

$$N_3 = [N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1] \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

Noise figure of a cascaded system





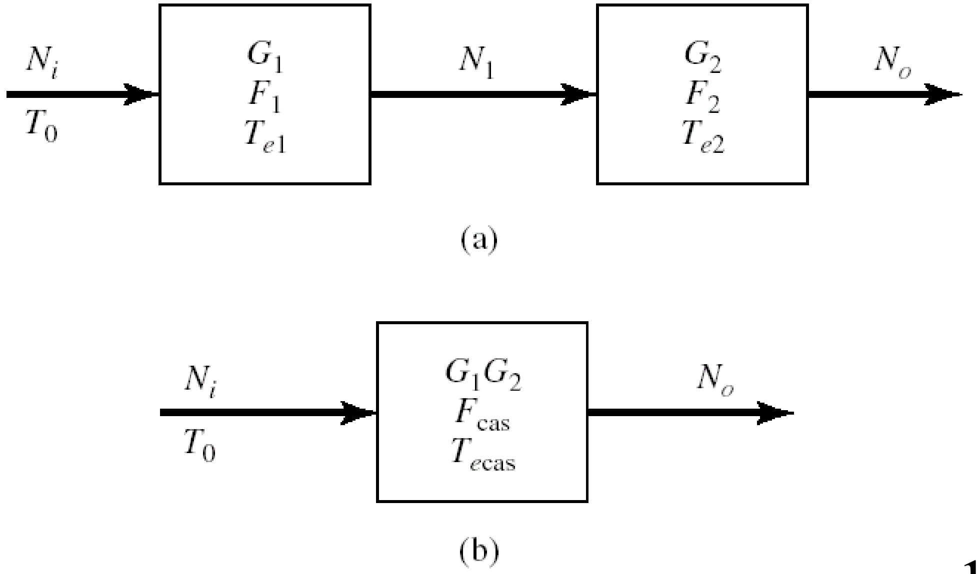
$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$G_{cas} = G_1 \cdot G_2 \qquad N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$

$$(F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2 = (F_{cas} - 1) \cdot N_0 \cdot G_1 \cdot G_2$$

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1)$$

Noise figure of a cascaded system



$$G_{cas} = G_1 \cdot G_2 \qquad F_{cas} = F_1 + \frac{1}{G_1}\left(F_2 - 1\right)$$

- Friis Formula (**!linear scale**)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

Friis Formula (noise)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

- Friis Formula shows that:
  - the overall noise figure of a cascaded system is largely determined by the noise characteristics of the first stage
  - the noise introduced by the following stages is reduced:
    - -1
    - division by G (usually G > 1)

Friis Formula (noise)

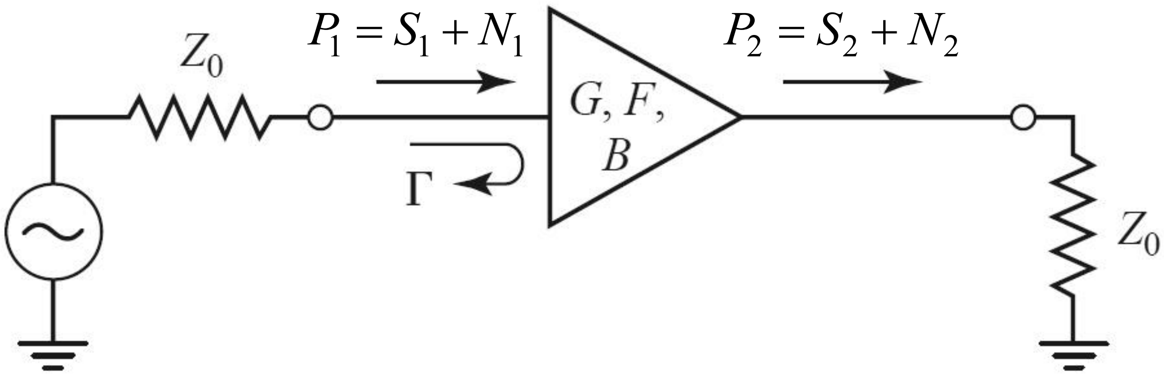
$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

- Effects of Friis Formula:
- in multi stage amplifiers:
  - it’s essential that the first stage is as noiseless as possible even if that means sacrificing power gain
  - the following stages can be optimized for power gain
- in single stage amplifiers:
  - in the input matching circuit it’s important to have noiseless elements (pure reactance, lossless lines)
  - output matching circuit has less influence on the noise (noise generated at this level appears when the desired signal has already been amplified by the transistor)

$$V_{n(ef)} = \sqrt{4kTBR} \qquad P_n = kTB$$

Noise Figure of a Mismatched Amplifier

- An input mismatched amplifier($\Gamma \neq 0$)



$$G_{ech} = G \cdot \left(1 - |\Gamma|^2\right)$$

$$N_2 = N_1 \cdot G \cdot \left(1 - |\Gamma|^2\right) + (F - 1) \cdot N_0 \cdot G = N_1 \cdot G \cdot \left(1 - |\Gamma|^2\right) + \frac{F - 1}{1 - |\Gamma|^2} \cdot N_0 \cdot G \cdot \left(1 - |\Gamma|^2\right)$$

$$N_2 = N_1 \cdot G_{ech} + (F_{ech} - 1) \cdot N_0 \cdot G_{ech}$$

$$F_{ech} = 1 + \frac{F - 1}{1 - |\Gamma|^2} \geq F$$

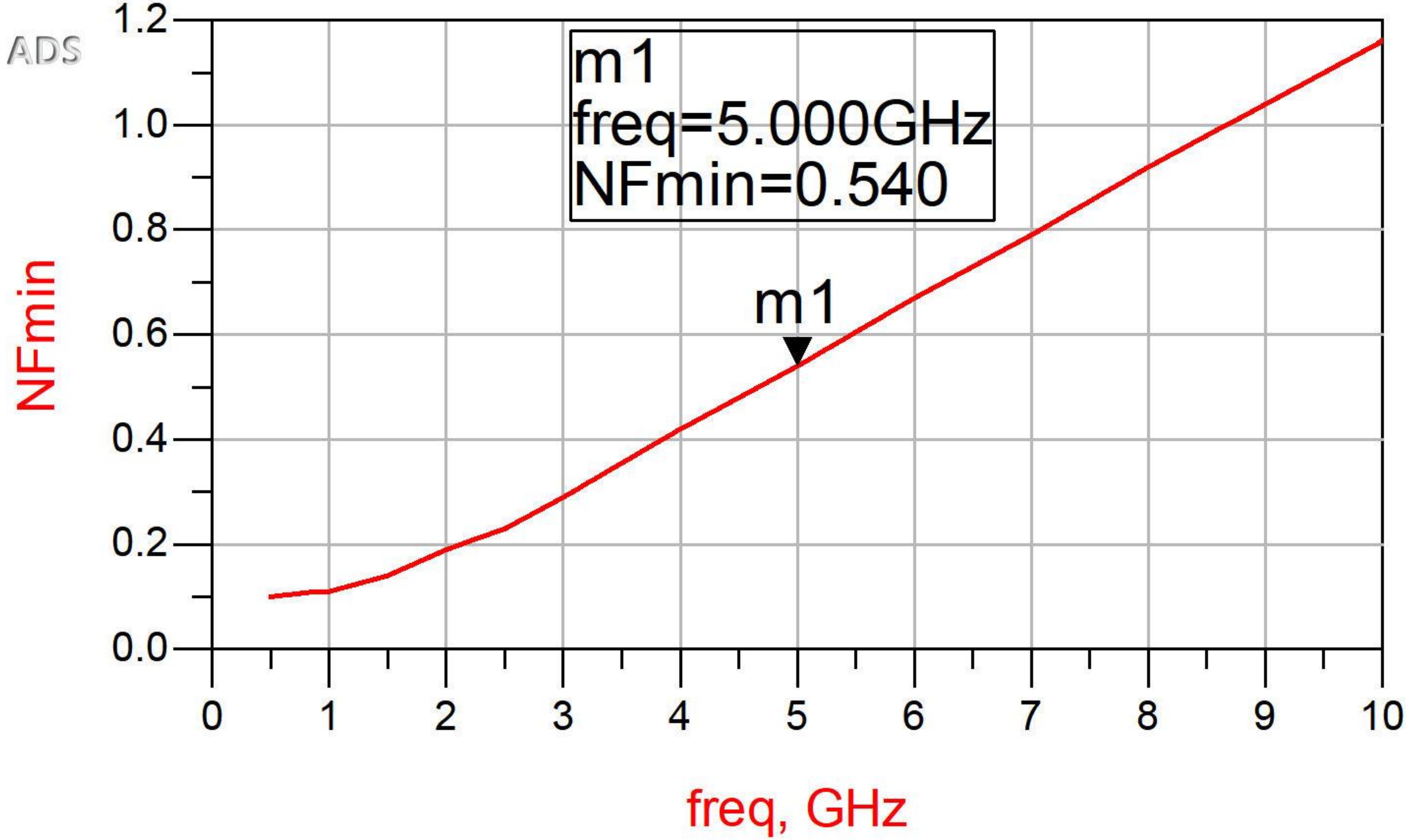
- Good noise figure **requires** good impedance matching

Example

- ATF-34143 **at Vds=3V Id=20mA.**
- @5GHz
  - $S_{11} = 0.64\angle 139°$
  - $S_{12} = 0.119\angle -21°$
  - $S_{21} = 3.165\angle 16°$
  - $S_{22} = 0.22\angle 146°$
  - $F_{min} = 0.54$ (**tipic [dB]**)
  - $\Gamma_{opt} = 0.45\angle 174°$
  - $r_n = 0.03$

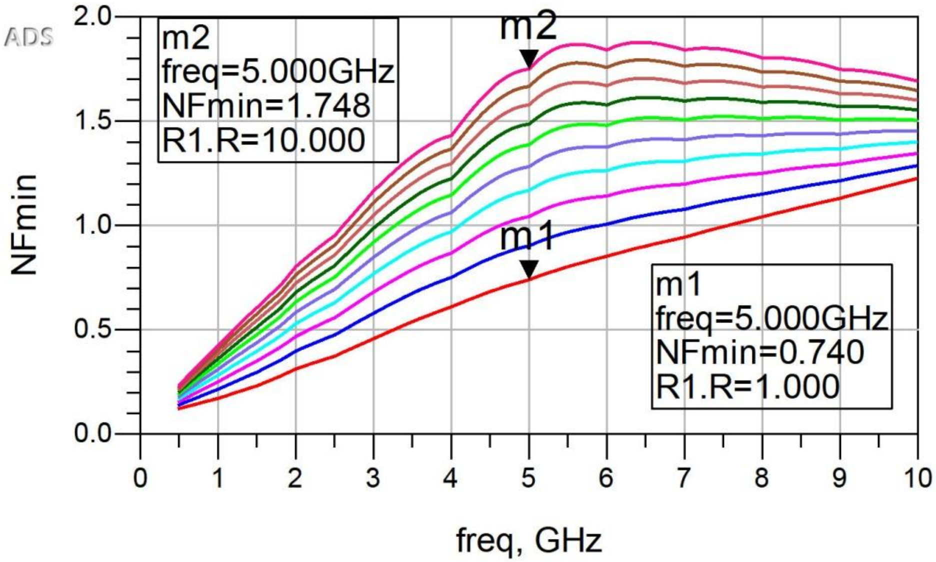
```
!ATF-34143
!S-PARAMETERS at Vds=3V  Id=20mA.   LAST UPDATED 01-29-99

# ghz s ma r 50

2.0  0.75  -126  6.306  90  0.088  23  0.26  -120
2.5  0.72  -145  5.438  75  0.095  15  0.25  -140
3.0  0.69  -162  4.762  62  0.102  7  0.23  -156
4.0  0.65  166  3.806  38  0.111  -8  0.22  174
5.0  0.64  139  3.165  16  0.119  -21  0.22  146
6.0  0.65  114  2.706  -5  0.125  -35  0.23  118
7.0  0.66  89  2.326  -27  0.129  -49  0.25  91
8.0  0.69  67  2.017  -47  0.133  -62  0.29  67
9.0  0.72  48  1.758  -66  0.135  -75  0.34  46

!FREQ   Fopt   GAMMA OPT      RN/Zo
!GHZ    dB     MAG     ANG     -

2.0  0.19  0.71  66  0.09
2.5  0.23  0.65  83  0.07
3.0  0.29  0.59  102  0.06
4.0  0.42  0.51  138  0.03
5.0  0.54  0.45  174  0.03
6.0  0.67  0.42  -151  0.05
7.0  0.79  0.42  -118  0.10
8.0  0.92  0.45  -88  0.18
9.0  1.04  0.51  -63  0.30
10.0  1.16  0.61  -43  0.46
```

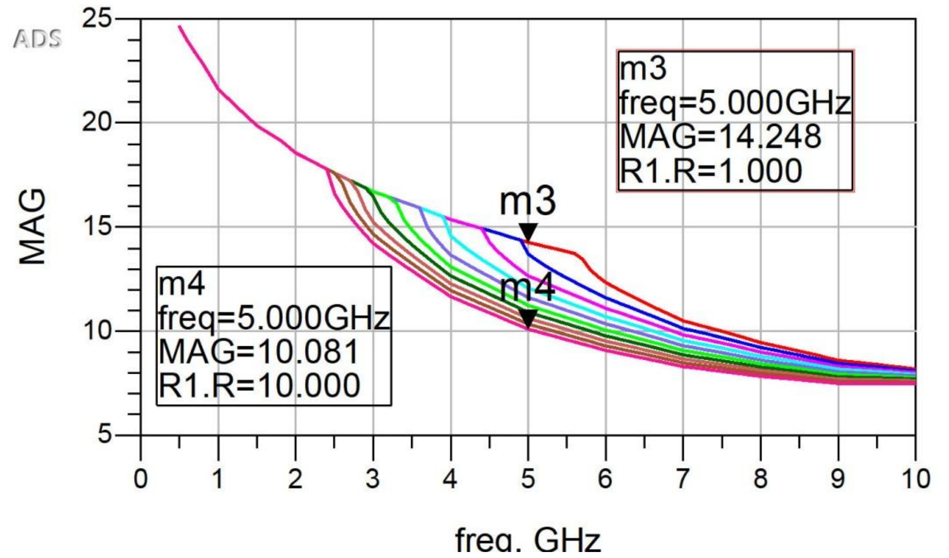
Example



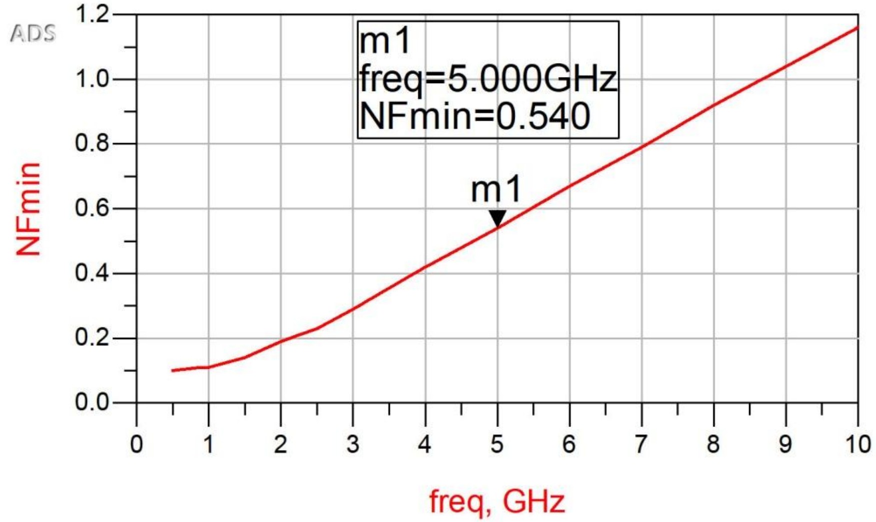
ADS
1.2
1.0
0.8
0.6
0.4
0.2
0.0
NFmin
m1
freq=5.000GHz
NFmin=0.540
m1
0
1
2
3
4
5
6
7
8
9
10
freq, GHz

Stabilization, input series resistor

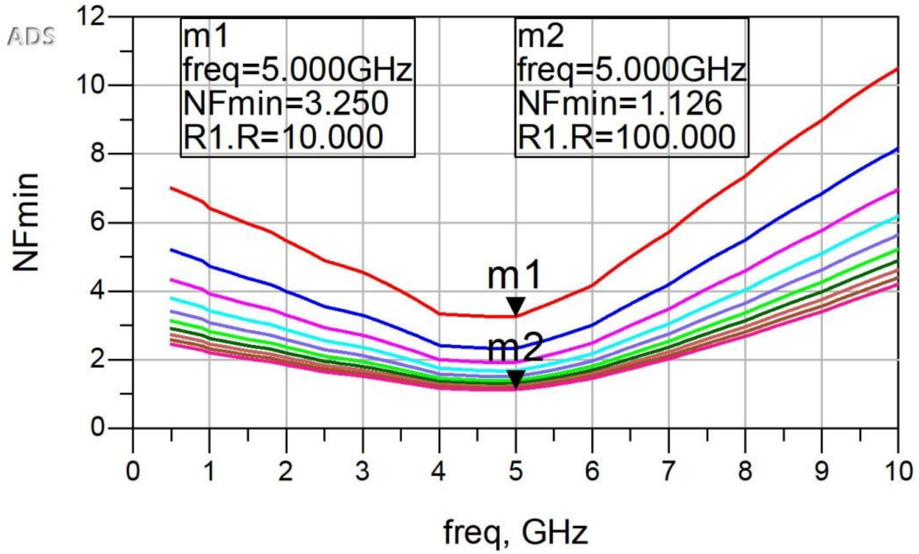


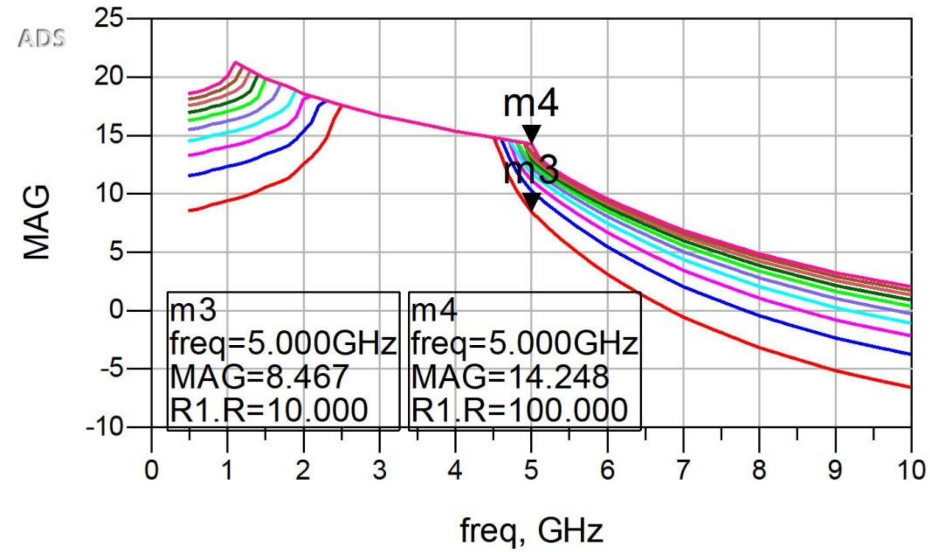


$$R_{SS} = 1 \div 10\ \Omega$$

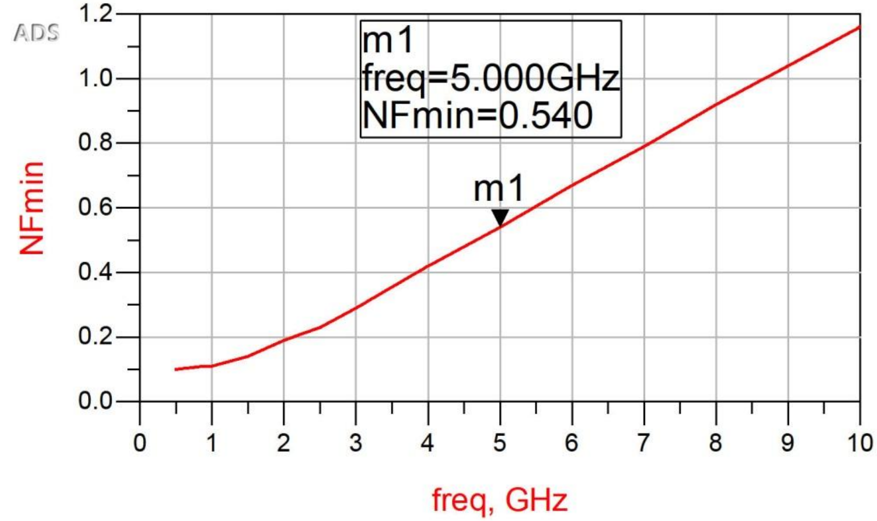


Stabilization, input shunt resistor

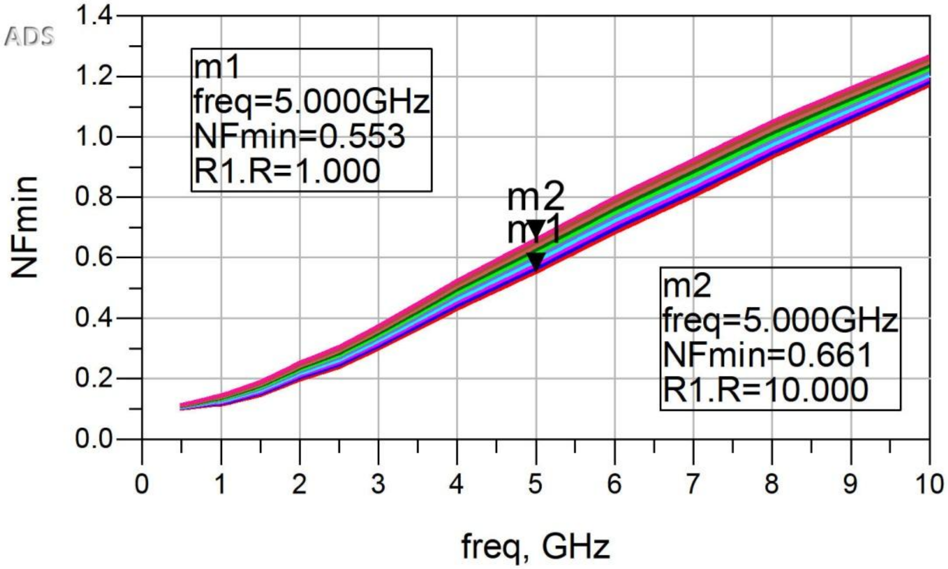


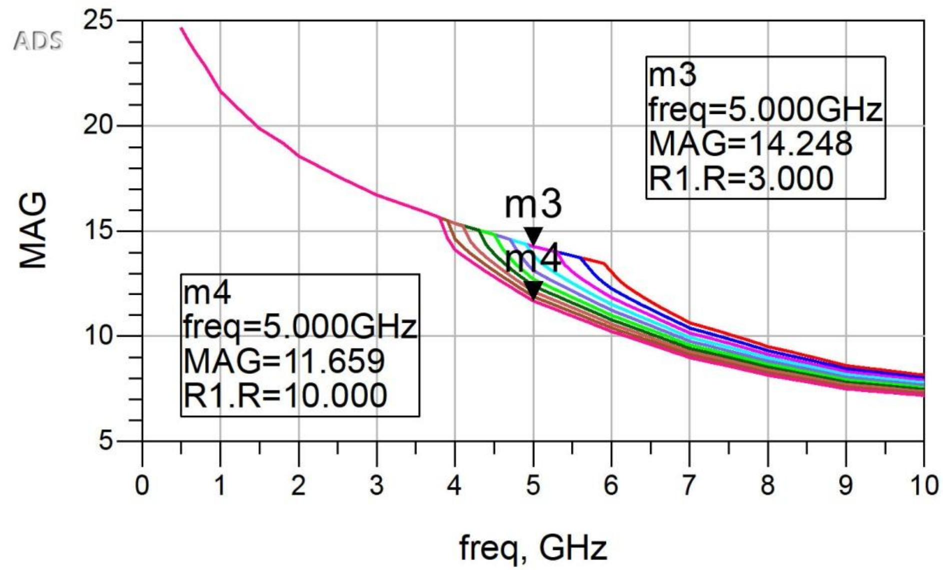


$$R_{PS} = 10 \div 100\,\Omega$$

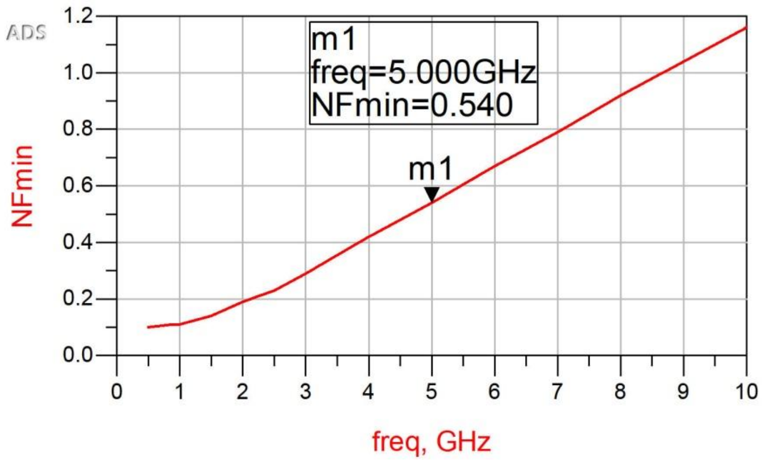


Stabilization, output series resistor

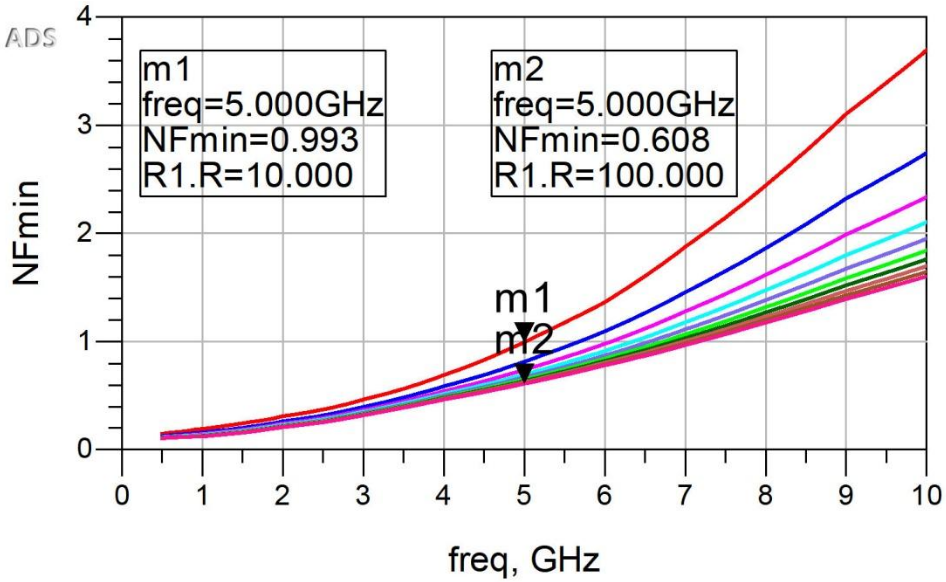


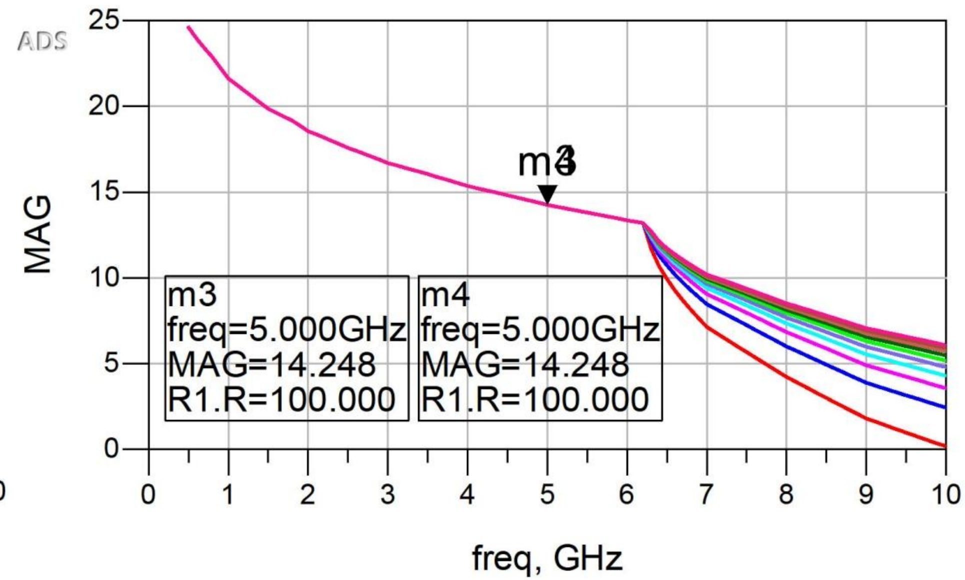


$$R_{SL} = 1 \div 10\ \Omega$$

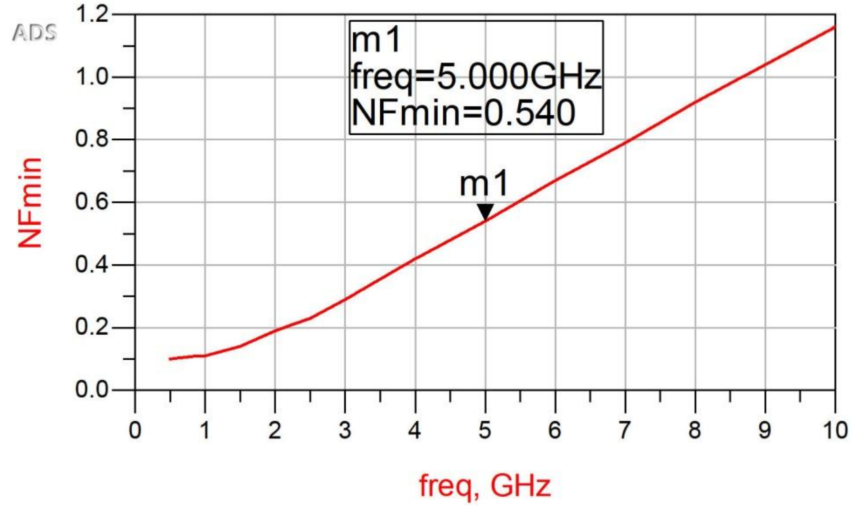


Stabilization, output shunt resistor





$$R_{PL} = 10 \div 100\ \Omega$$



Noise figure of a two-port amplifier

- 3 noise parameters (2reals + 1 complex):

$$F_{\min},\, r_n = \frac{R_N}{Z_0},\, \Gamma_{opt}$$

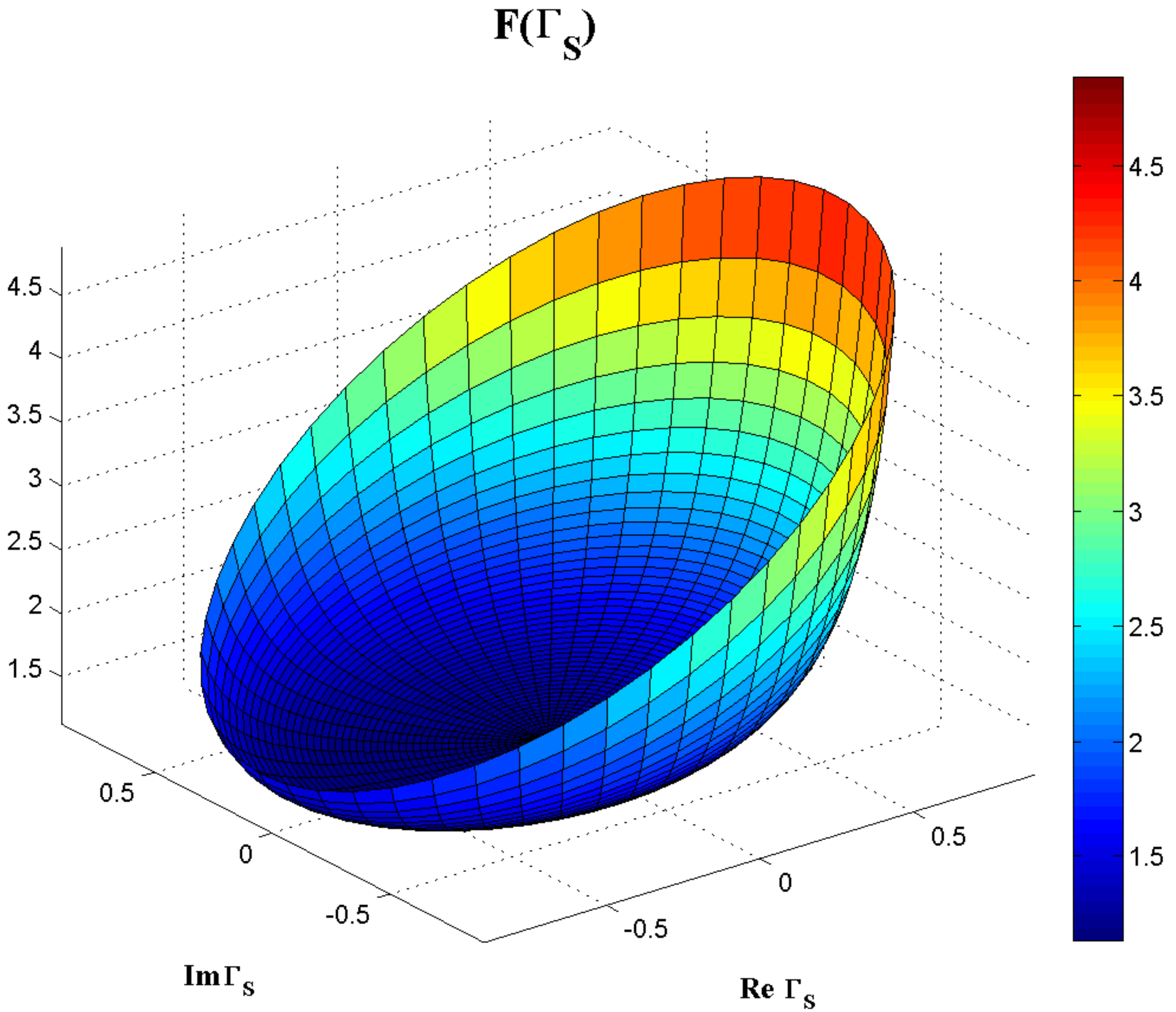
$$F = F_{\min} + \frac{R_N}{G_S} \cdot \left| Y_S - Y_{opt} \right|^2 \qquad Y_S = \frac{1}{Z_0} \cdot \frac{1-\Gamma_S}{1+\Gamma_S} \qquad Y_{opt} = \frac{1}{Z_0} \cdot \frac{1-\Gamma_{opt}}{1+\Gamma_{opt}}$$

$$F = F_{\min} + 4 \cdot r_n \cdot \frac{\left| \Gamma_S - \Gamma_{opt} \right|^2}{\left(1 - \left| \Gamma_S \right|^2\right) \cdot \left| 1 + \Gamma_{opt} \right|^2}$$

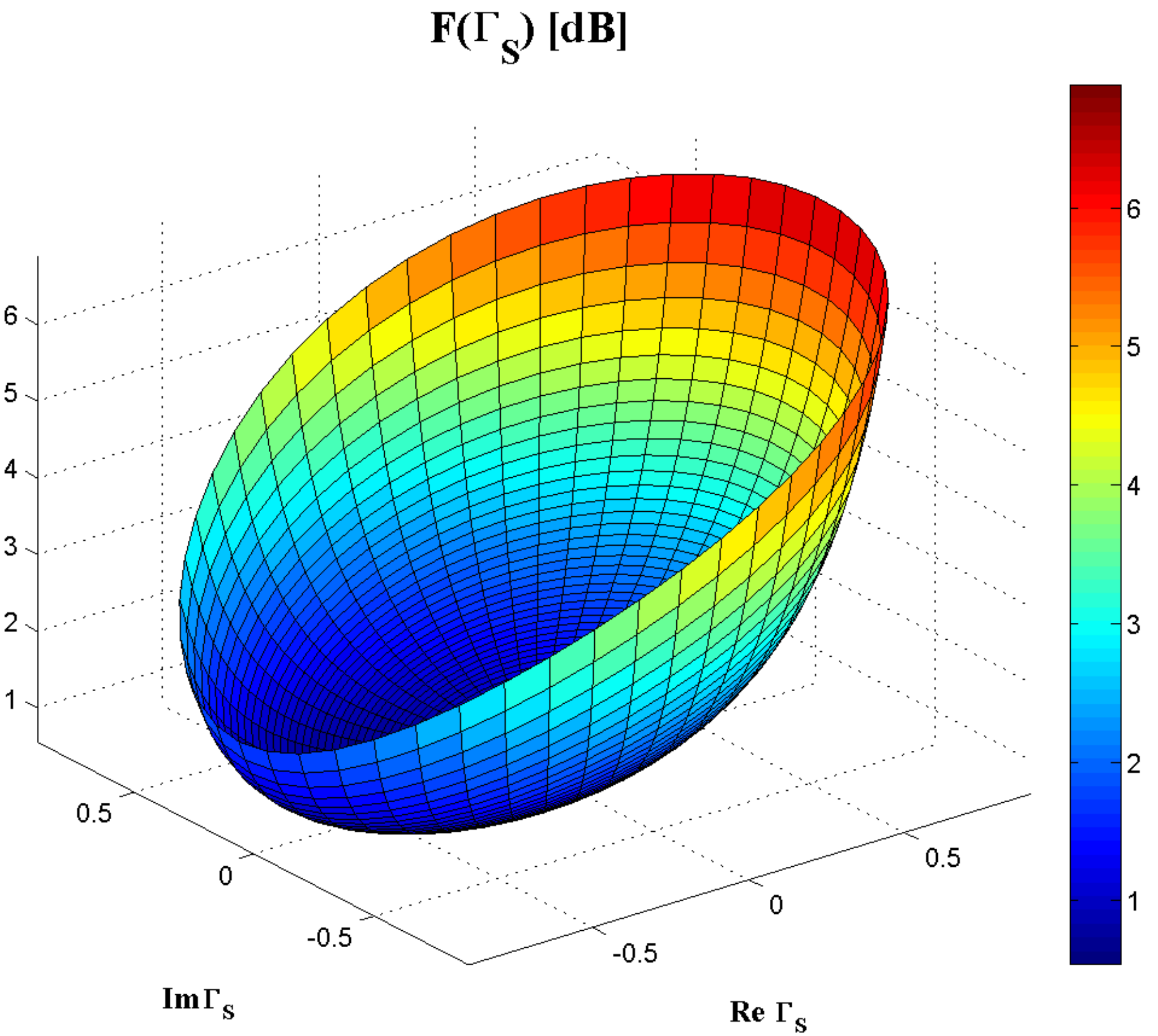
- Γ_{opt} optimum source reflection coefficient that results in minimum noise figure

$$\Gamma_S = \Gamma_{opt} \Rightarrow F = F_{\min}$$

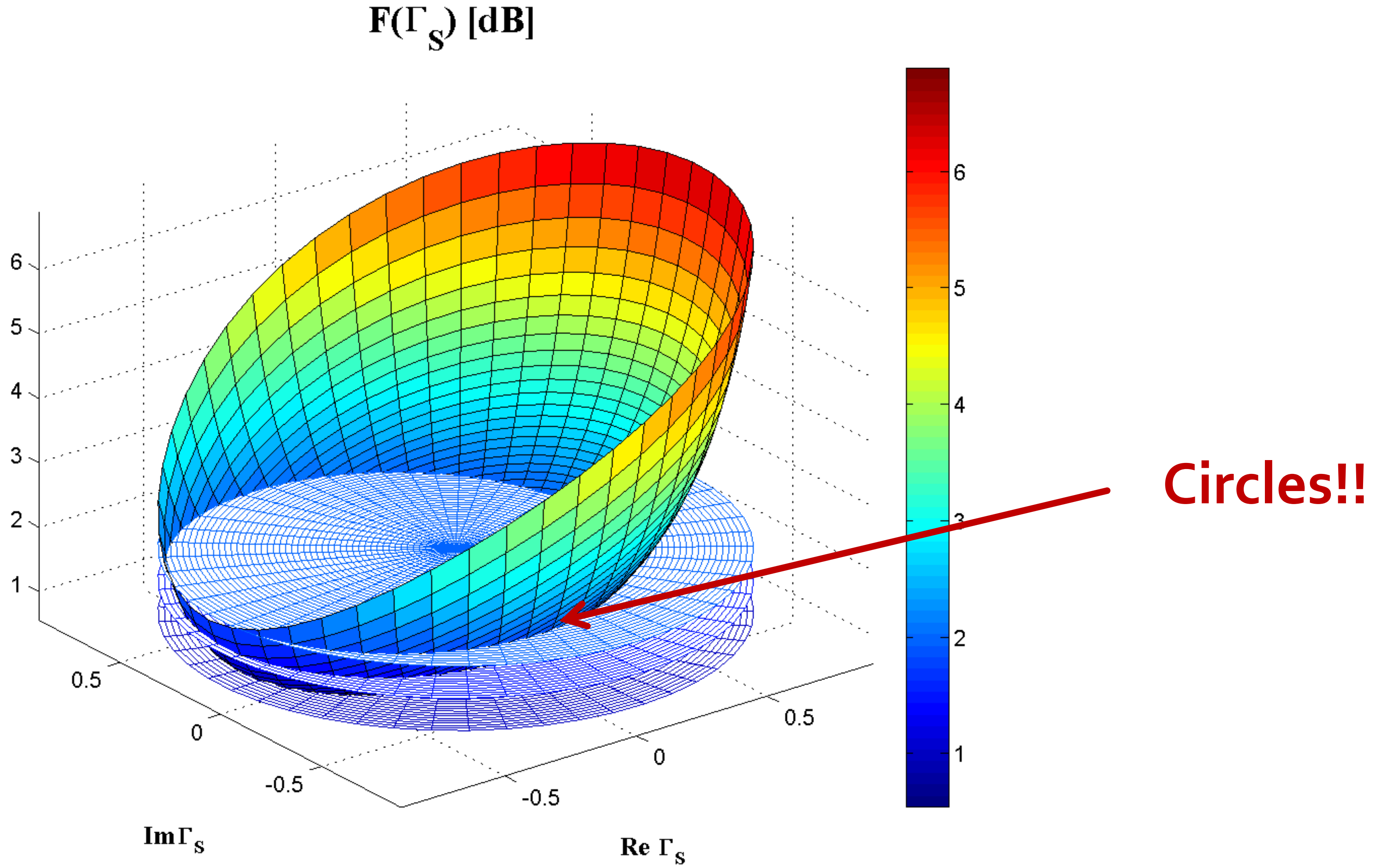
$F(\Gamma_S)$



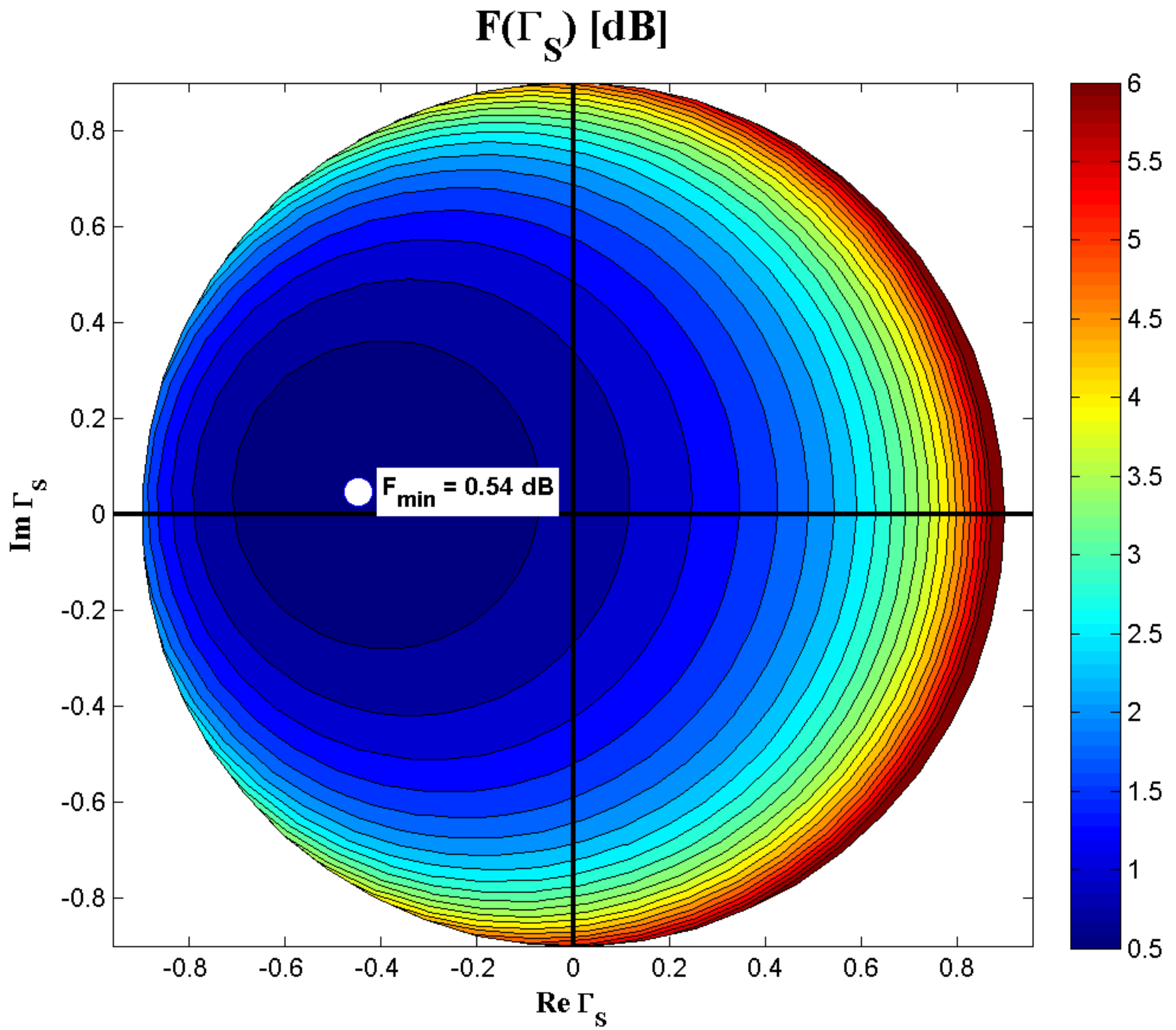
$F[dB](\Gamma_S)$



F[dB](Γ_S), constant value contours



$G_S[dB](\Gamma_S)$, constant value contours



$\Gamma opt = 0.45 \angle 174°$

Circles of constant noise figure

$$F = F_{\min} + 4 \cdot r_n \cdot \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{\left(1 - \left|\Gamma_S\right|^2\right) \cdot \left|1 + \Gamma_{opt}\right|^2}$$

- We define N (noise figure parameter)
  - **N** constant for **F** constant

$$N = \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{1 - \left|\Gamma_S\right|^2} = \frac{F - F_{\min}}{4 \cdot r_n} \cdot \left|1 + \Gamma_{opt}\right|^2$$

$$\left(\Gamma_S - \Gamma_{opt}\right) \cdot \left(\Gamma_S^* - \Gamma_{opt}^*\right) = N \cdot \left(1 - \left|\Gamma_S\right|^2\right)$$

$$\Gamma_S \cdot \Gamma_S^* + N \cdot \left|\Gamma_S\right|^2 - \left(\Gamma_S \cdot \Gamma_{opt}^* - \Gamma_S^* \cdot \Gamma_{opt}\right) + \Gamma_{opt} \cdot \Gamma_{opt}^* = N$$

$$\Gamma_S \cdot \Gamma_S^* - \frac{\Gamma_S \cdot \Gamma_{opt}^* - \Gamma_S^* \cdot \Gamma_{opt}}{N+1} + \Gamma_{opt} \cdot \Gamma_{opt}^* = \frac{N - \left|\Gamma_{opt}\right|^2}{N+1} + \frac{\left|\Gamma_{opt}\right|^2}{(N+1)^2}$$

$$|a+b|^2 = (a+b) \cdot (a+b)^* = (a+b) \cdot \left(a^* + b^*\right) = |a|^2 + |b|^2 + a^* \cdot b + a \cdot b^*$$

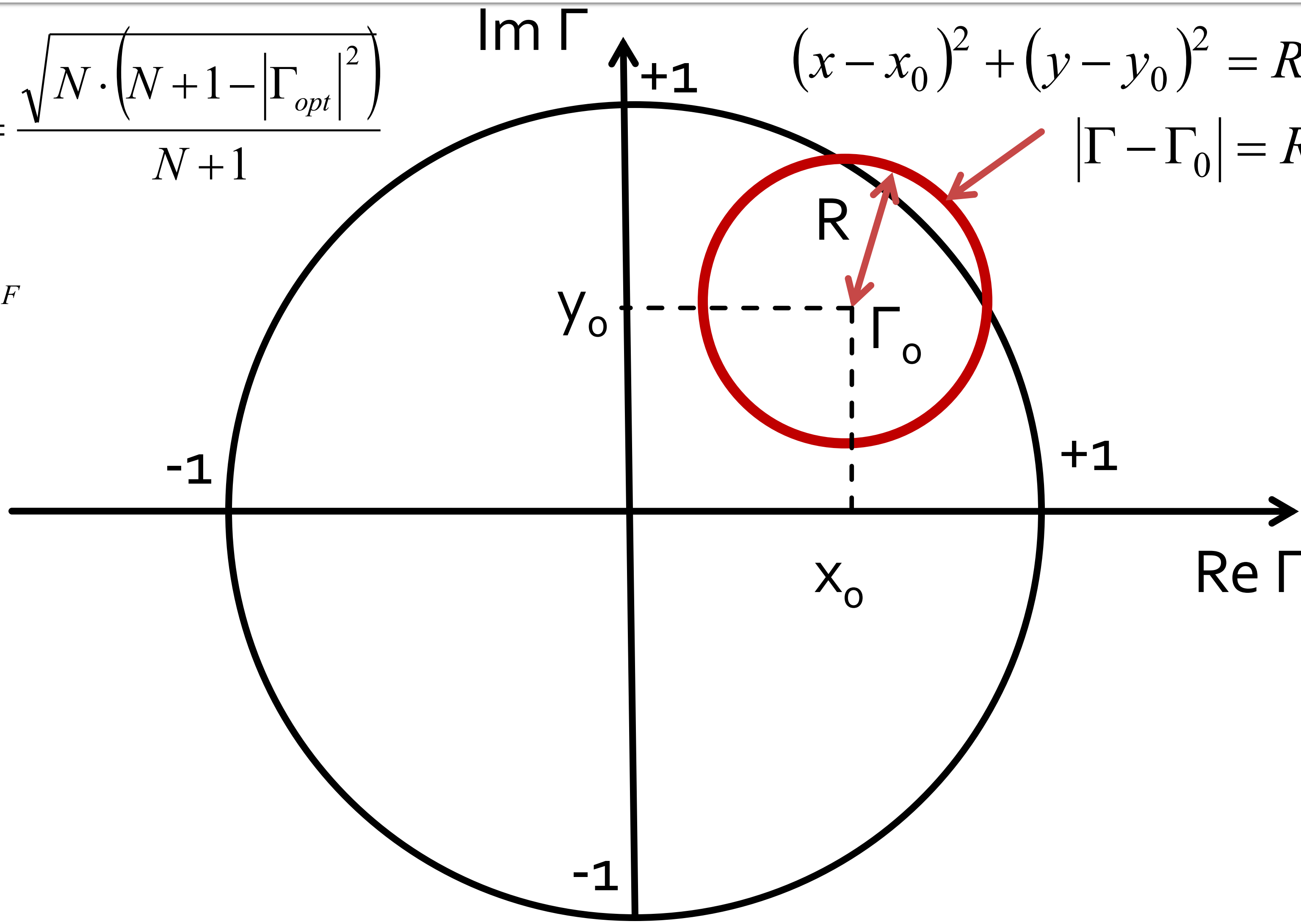
Circles of constant noise figure

$$\left|\Gamma_S - \frac{\Gamma_{opt}}{N+1}\right| = \frac{\sqrt{N\cdot\left(N+1-\left|\Gamma_{opt}\right|^2\right)}}{N+1}$$

$$\left|\Gamma_S - C_F\right| = R_F$$

$$(x-x_0)^2 + (y-y_0)^2 = R^2$$

$$\left|\Gamma - \Gamma_0\right| = R$$



Circles of constant noise figure

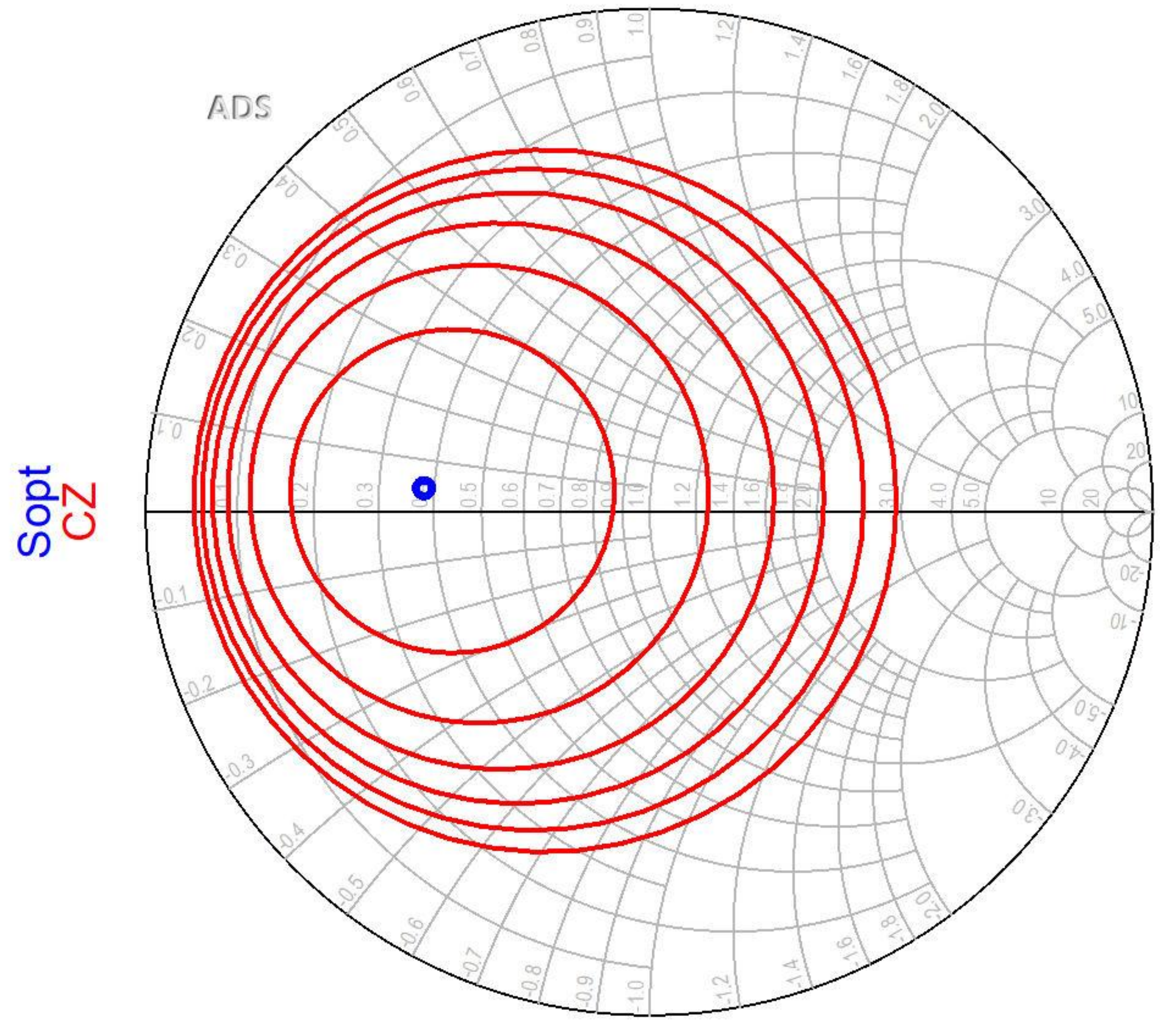
$$N = \frac{F - F_{\min}}{4 \cdot r_n} \cdot \left|1 + \Gamma_{opt}\right|^2$$

$$\left|\Gamma_S - \frac{\Gamma_{opt}}{N+1}\right| = \frac{\sqrt{N \cdot \left(N + 1 - \left|\Gamma_{opt}\right|^2\right)}}{N+1}$$

$$\left|\Gamma_S - C_F\right| = R_F \qquad C_F = \frac{\Gamma_{opt}}{N+1} \qquad R_F = \frac{\sqrt{N \cdot \left(N + 1 - \left|\Gamma_{opt}\right|^2\right)}}{N+1}$$

- The locus in the complex plane Γ_S of the points with constant noise figure is a circle
- **Interpretation**: Any reflection coefficient Γ_S which plotted in the complex plane lies **on** the circle drawn for F_{circle} will lead to a noise factor $F = F_{circle}$
  - Any reflection coefficient Γ_S plotted **outside** this circle will lead to a noise factor $F > F_{circle}$
  - Any reflection coefficient Γ_S plotted **inside** this circle will lead to a noise factor $F < F_{circle}$

ADS

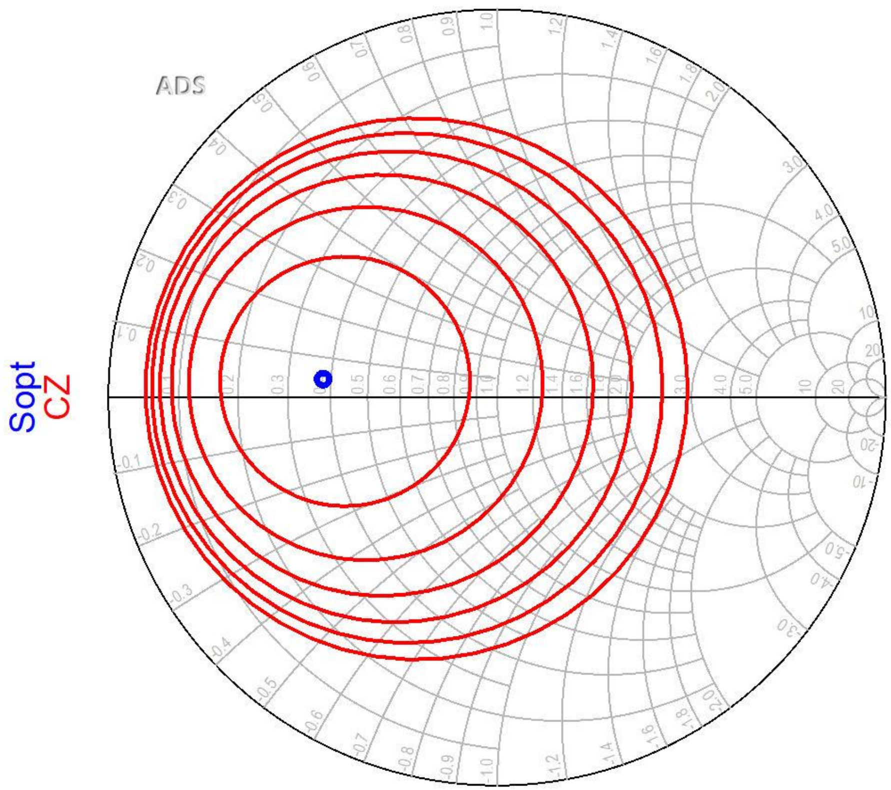


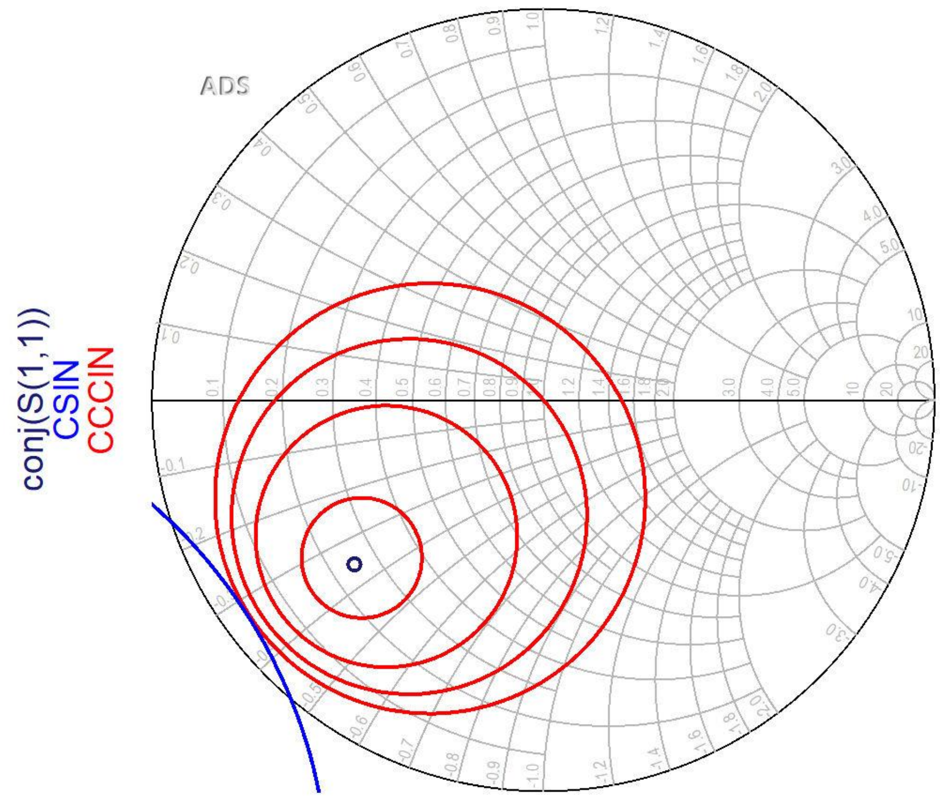
Circles of constant noise figure

- The noise internally generated by the transistor depends **only** by the input matching circuit
- A minimum noise figure is possible (NF_{min} – a datasheet/"s2p file" parameter for the transistor)
- If we design a low noise amplifier (**LNA**) the usual design technique is as follows:
  - design of the input matching circuit solely (largely) for noise optimization
  - design of output matching circuit for gain compensation/optimization (if lossy circuits are used the output matching circuit noise can be added but the transistor noise is not influenced)

LNA – Low Noise Amplifier

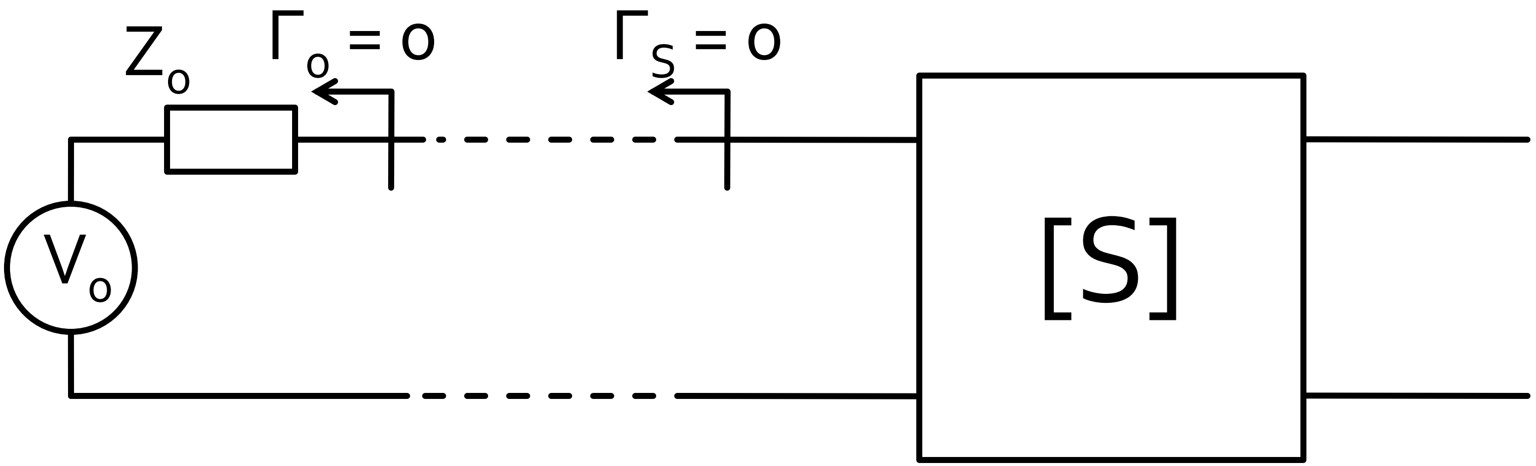
- Usually a transistor suitable for implementing an LNA at a certain frequency will have input gain circles and noise circles in the same area for Γ_S





Matching – 1

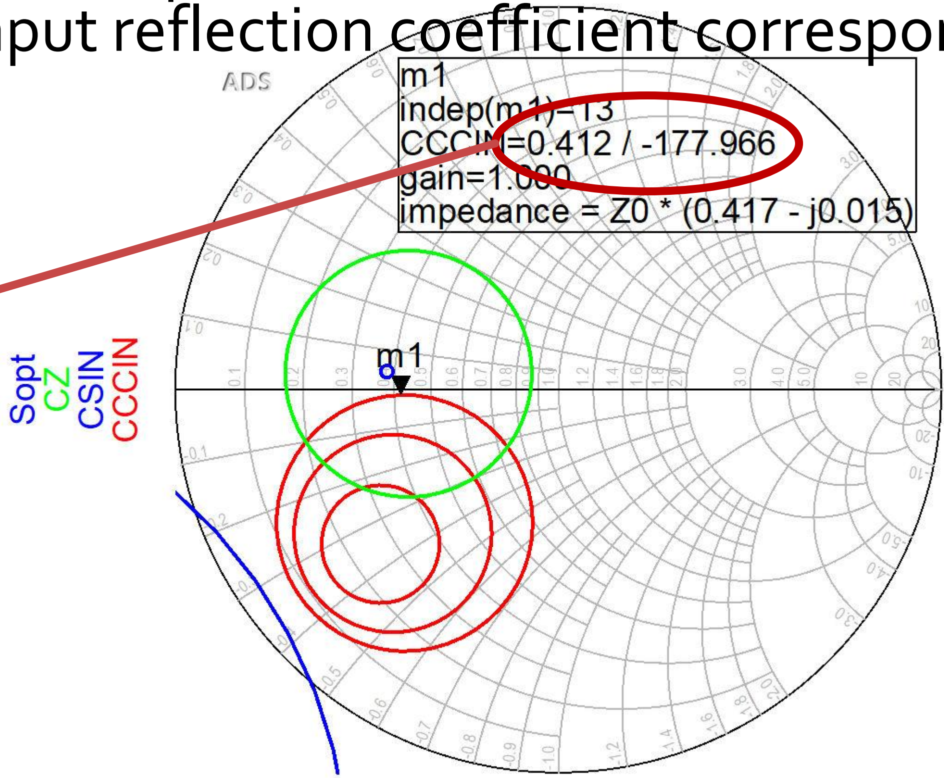
- Connecting the amplifier (transistor) directly to the source with Zo generate a reflection coefficient seen towards the source equal with **0** (complex number, $\Gamma_0 = 0 + 0 \cdot j$)
  - most of the time this reflection coefficient does not offer optimum noise/gain



Matching – 2

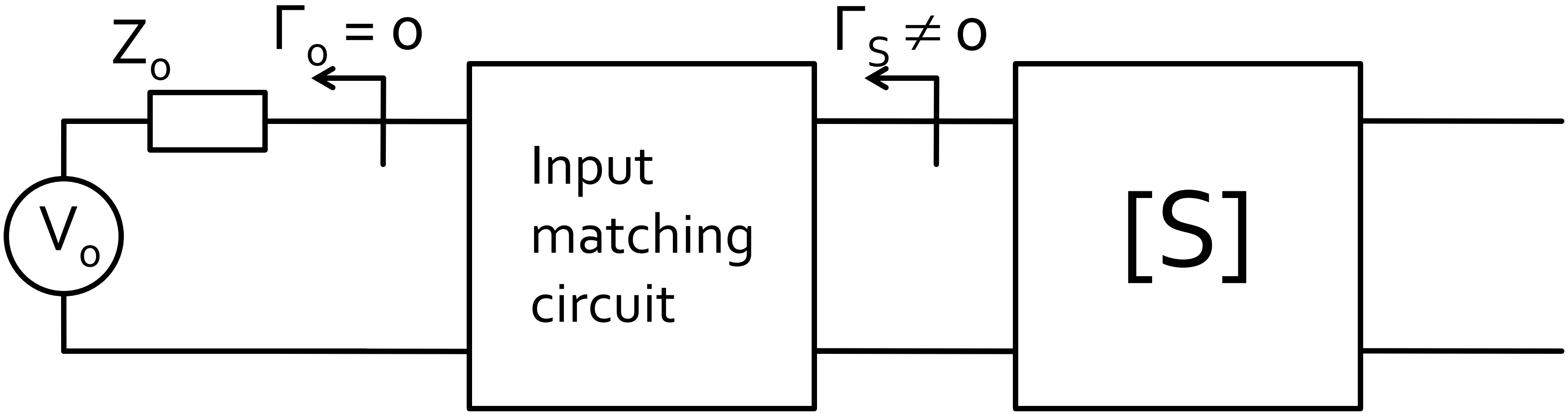
- We plot on the complex plane (Smith Chart) the stability/gain/noise circles (depending on the particular application)
- We choose a point with a suitable position relative to these circles (also application dependent)
- We determine the input reflection coefficient corresponding to this point, Γ_S

$$\Gamma_S = 0.412\angle -177.966°$$



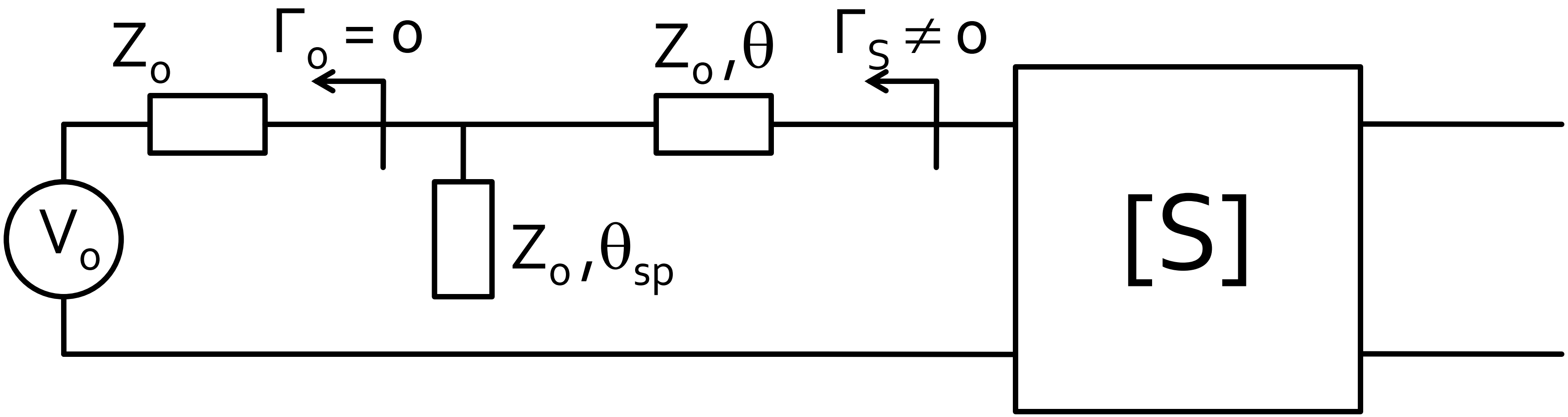
Matching – 3

- We insert the input matching circuits which allows the transistor to see towards the source the previously determined reflection coefficient Γ_S



Matching – 4

- Easiest to design matching section consists in the insertion of (in order from the transistor towards the Z_o source):
  - a series Z_o line, with electrical length θ
  - a shunt stub, open-circuited, made from a Z_o line, with electrical length θ_{sp}



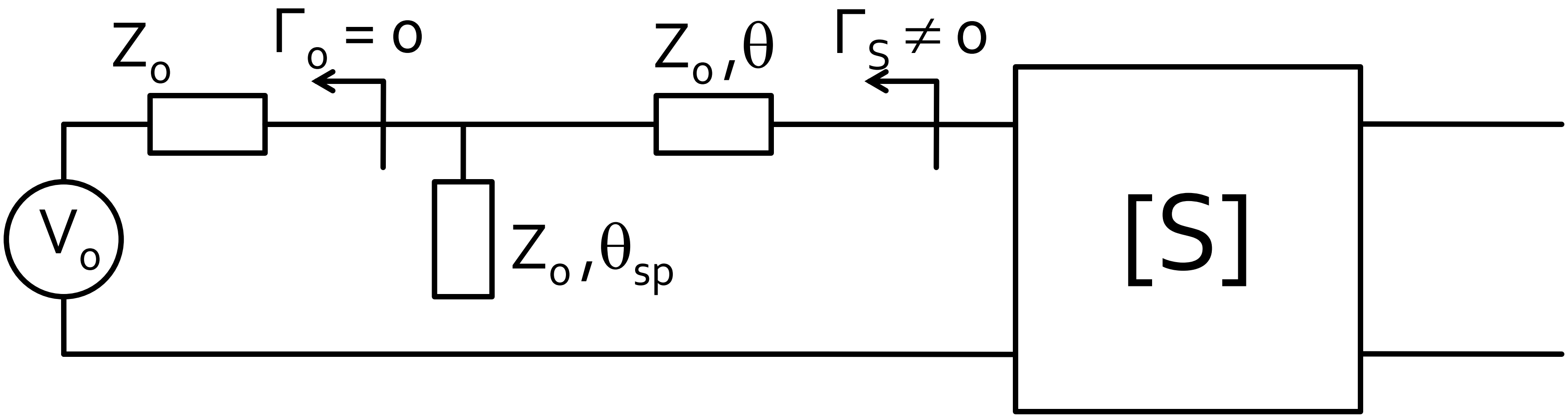
Matching – 5

- Computation depends solely on Γ_S (magnitude and phase)

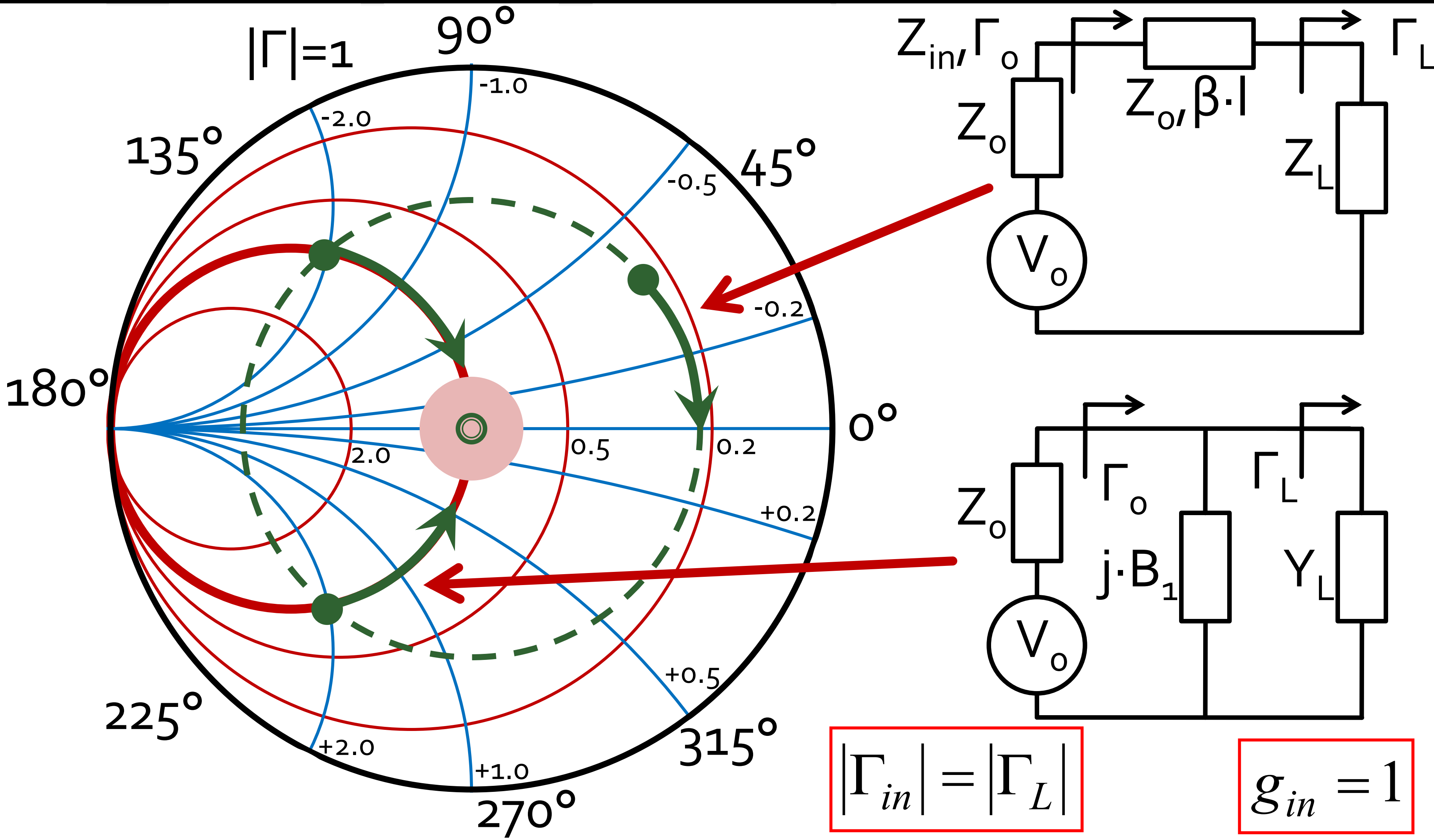
$$\cos(\varphi_S + 2\theta) = -|\Gamma_S|$$

$$\tan\theta_{sp} = \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

- The sign (+/-) chosen for the series line equation imposes the sign used for the shunt stub equation



Shunt stub matching, L8



$$|\Gamma_{in}| = |\Gamma_L|$$

$$g_{in} = 1$$

Example, LNA @ 5 GHz

- ATF-34143 **at Vds=3V Id=20mA.**
- @5GHz
  - $S_{11} = 0.64 \angle 139°$
  - $S_{12} = 0.119 \angle -21°$
  - $S_{21} = 3.165 \angle 16°$
  - $S_{22} = 0.22 \angle 146°$
  - $F_{min} = 0.54$ (**tipic [dB]**)
  - $\Gamma_{opt} = 0.45 \angle 174°$
  - $r_n = 0.03$

```
!ATF-34143
!S-PARAMETERS at Vds=3V  Id=20mA.   LAST UPDATED 01-29-99

# ghz s ma r 50

2.0  0.75  -126  6.306  90  0.088  23  0.26  -120
2.5  0.72  -145  5.438  75  0.095  15  0.25  -140
3.0  0.69  -162  4.762  62  0.102  7  0.23  -156
4.0  0.65  166  3.806  38  0.111  -8  0.22  174
5.0  0.64  139  3.165  16  0.119  -21  0.22  146
6.0  0.65  114  2.706  -5  0.125  -35  0.23  118
7.0  0.66  89  2.326  -27  0.129  -49  0.25  91
8.0  0.69  67  2.017  -47  0.133  -62  0.29  67
9.0  0.72  48  1.758  -66  0.135  -75  0.34  46

!FREQ   Fopt    GAMMA OPT       RN/Zo
!GHZ     dB     MAG     ANG      -

2.0  0.19  0.71  66  0.09
2.5  0.23  0.65  83  0.07
3.0  0.29  0.59  102  0.06
4.0  0.42  0.51  138  0.03
5.0  0.54  0.45  174  0.03
6.0  0.67  0.42  -151  0.05
7.0  0.79  0.42  -118  0.10
8.0  0.92  0.45  -88  0.18
9.0  1.04  0.51  -63  0.30
10.0  1.16  0.61  -43  0.46
```

Example, LNA @ 5 GHz

- Low Noise Amplifier
- At the input matching a compromise is required between:
  - noise (~~input~~ constant noise figure circles)
  - gain (input constant gain circles)
  - stability (input stability circle)
- At the output matching noise **is not influenced**. A compromise is required between :
  - gain (output constant gain circles)
  - stability (output stability circle)

Example, LNA @ 5 GHz

$$U = \frac{|S_{12}|\cdot|S_{21}|\cdot|S_{11}|\cdot|S_{22}|}{\left(1-|S_{11}|^2\right)\cdot\left(1-|S_{22}|^2\right)} = 0.094$$

$$-0.783\,dB < G_T[dB] - G_{TU}[dB] < 0.861\,dB$$

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2}\cdot|S_{21}|^2\cdot\frac{1}{1-|S_{22}|^2} = 17.83$$

$$G_{TU\max}[dB] = 12.511\,dB$$

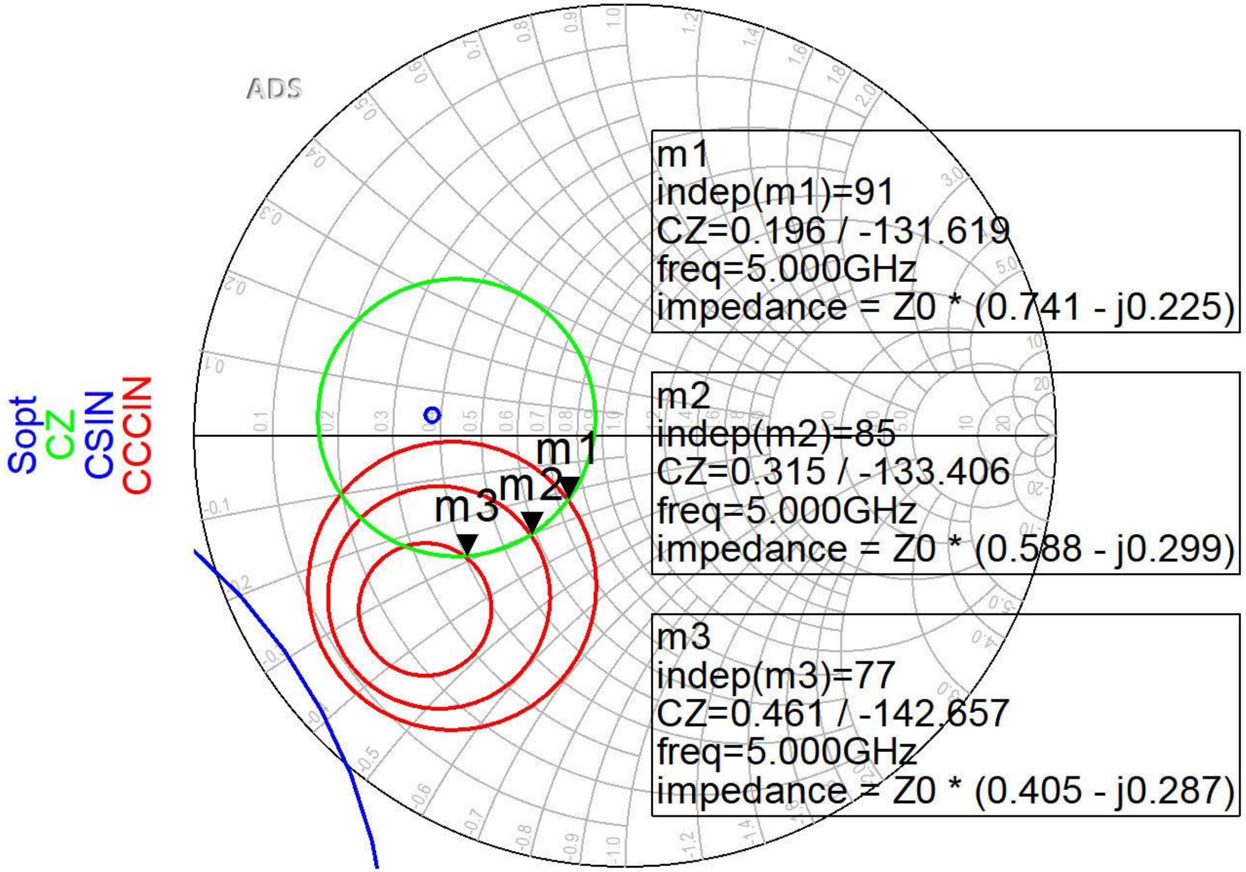
$$G_0 = |S_{21}|^2 = 10.017 = 10.007\,dB$$

$$G_{S\max} = \frac{1}{1-|S_{11}|^2} = 1.694 = 2.289\,dB$$

$$G_{L\max} = \frac{1}{1-|S_{22}|^2} = 1.051 = 0.215\,dB$$

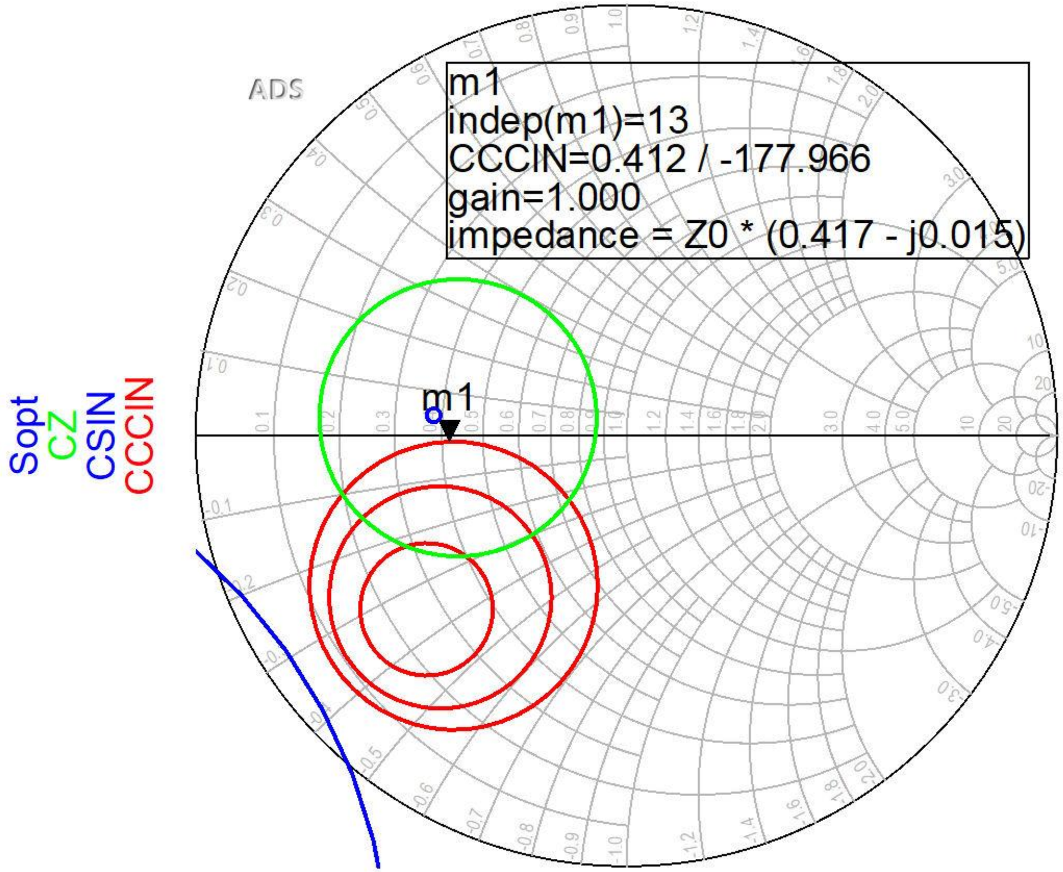
- In this particular case G_{Lmax} = 0.21 dB, the transistor could be used directly connected to the 50Ω load
- The absence of the output matching circuit **is not** recommended. While the attainable power gain is low, it's absence eliminates the possibility to use it to compensate an improper gain generated by the noise optimization of the input matching circuit

Input matching circuit



- For the input matching circuit
  - noise circle CZ: 0.75dB
  - input constant gain circles CCCIN: 1dB, 1.5dB, 2 dB
- We choose (small Q → wide bandwidth) position m1

Input matching circuit



- If we can afford a 1.2dB decrease of the input gain for better NF,Q (Gs = 1 dB), position m1 above is better
- We obtain better (smaller) NF

Input matching circuit

- Position m1 in complex plane (Smith Chart)

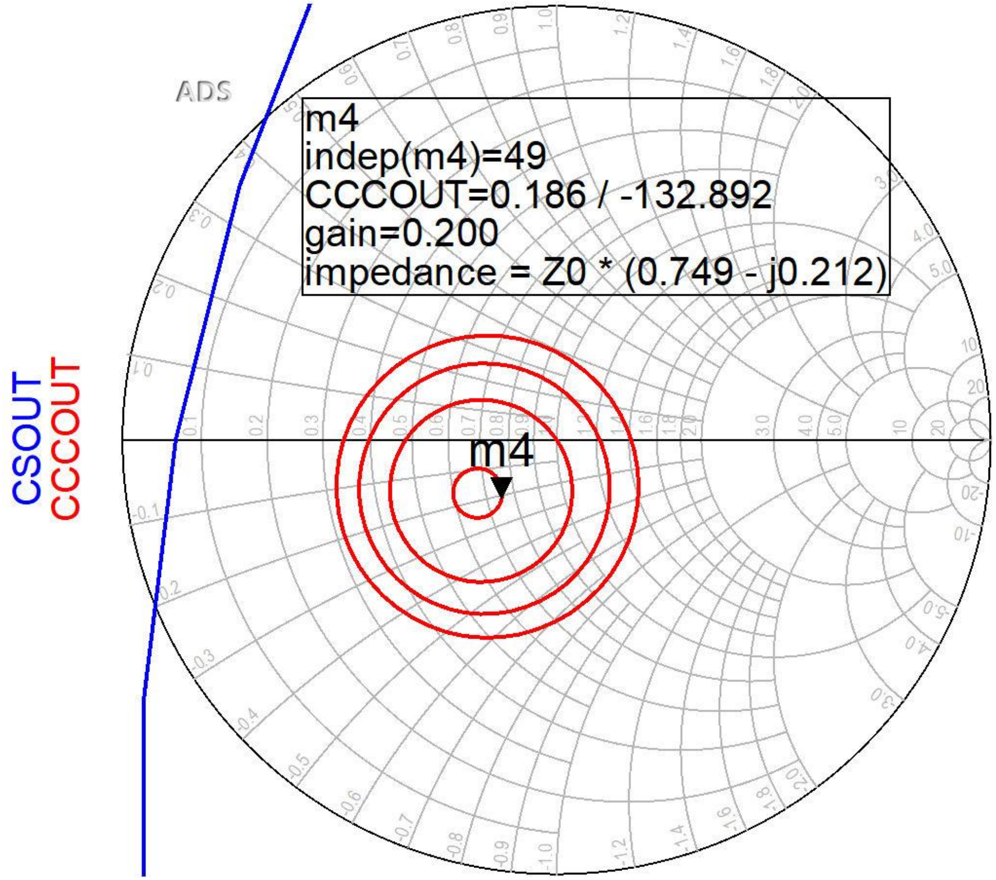
$$\Gamma_S = 0.412\angle -178° \qquad |\Gamma_S| = 0.412; \quad \varphi = -178°$$

$$\cos(\varphi + 2\theta) = -|\Gamma_S| \qquad \mathrm{Im}[y_S(\theta)] = \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\cos(\varphi + 2\theta) = -0.412 \Rightarrow \quad (\varphi + 2\theta) = \pm 114.33°$$

$$(\varphi + 2\theta) = \begin{cases} +114.33° \\ -114.33° \end{cases} \quad \theta = \begin{cases} 146.2° \\ 31.8° \end{cases} \quad \mathrm{Im}[y_S(\theta)] = \begin{cases} -0.904 \\ +0.904 \end{cases} \quad \theta_{sp} = \begin{cases} 137.9° \\ 42.1° \end{cases}$$

Output matching circuit



- output constant gain circles CCCOUT: -0.4dB, -0.2dB, 0dB, +0.2dB
- the lack of noise restrictions allows optimization for better gain (close to maximum – position m4)

Output matching circuit

- Position m4 in complex plane (Smith Chart)

$$\Gamma_L = 0.186\angle -132.9°$$

$$|\Gamma_L| = 0.186; \quad \varphi = -132.9°$$

$$\cos(\varphi + 2\theta) = -|\Gamma_L|$$

$$\mathrm{Im}[y_L(\theta)] = \frac{-2\cdot|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}} = -0.379$$

$$\cos(\varphi + 2\theta) = -0.186 \Rightarrow \quad (\varphi + 2\theta) = \pm 100.72°$$

$$(\varphi + 2\theta) = \begin{cases} +100.72° \\ -100.72° \end{cases} \quad \theta = \begin{cases} 116.8° \\ 16.1° \end{cases} \quad \mathrm{Im}[y_L(\theta)] = \begin{cases} -0.379 \\ +0.379 \end{cases} \quad \theta_{sp} = \begin{cases} 159.3° \\ 20.7° \end{cases}$$

LNA

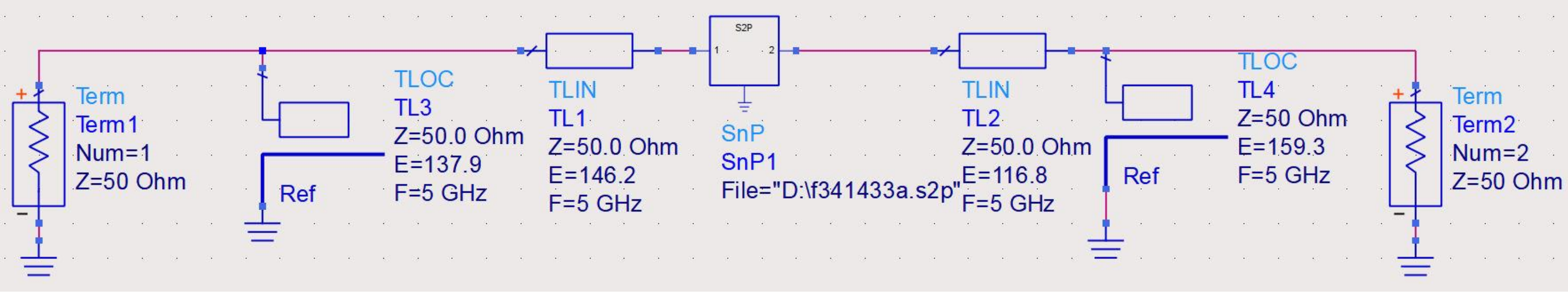
- We estimate a gain (in unilateral assumption, ±0.9 dB)

$$G_T[dB] = G_S[dB] + G_0[dB] + G_L[dB]$$

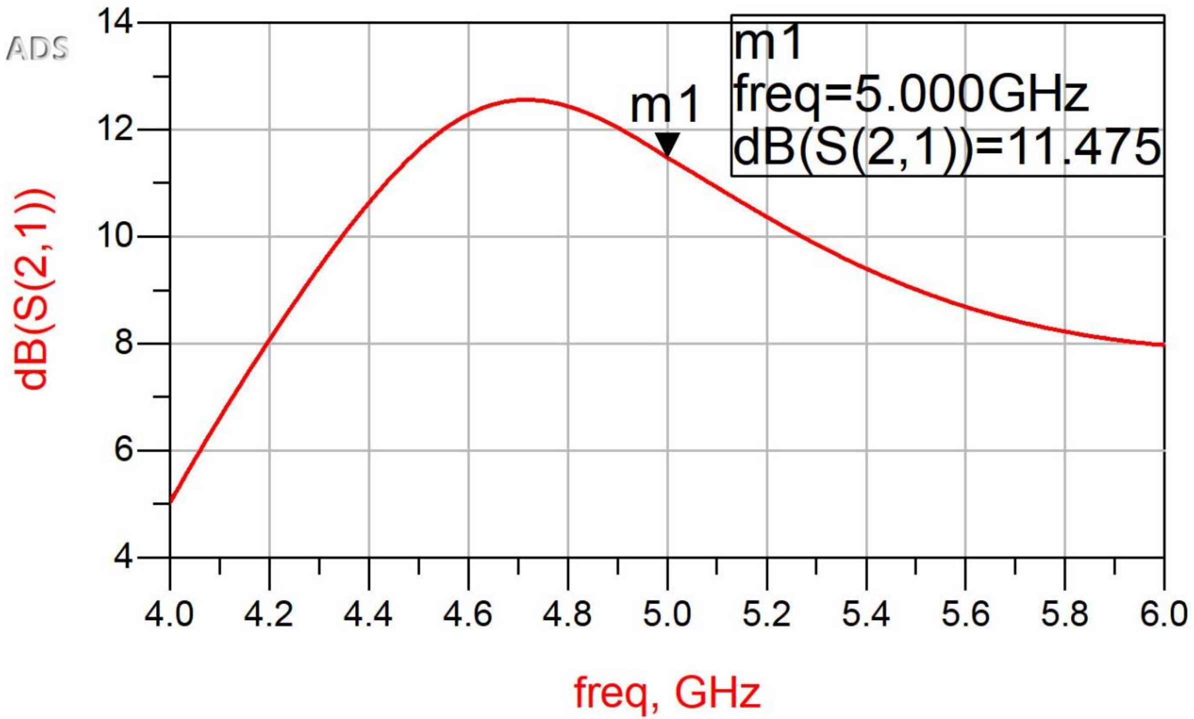
$$G_T[dB] = 1\ dB + 10\ dB + 0.2\ dB = 11.2\ dB$$

- We estimate a noise factor well bellow 0.75dB (quite close to the minimum ~0.6 dB)

ADS

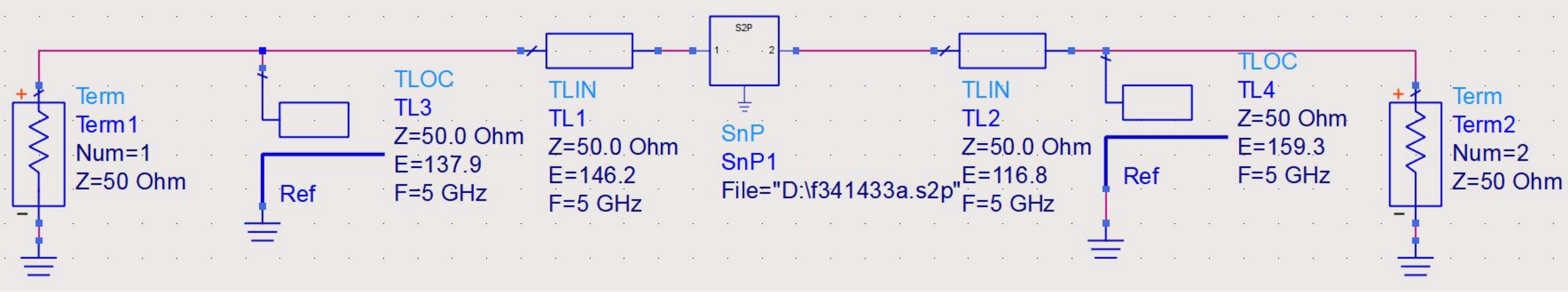


Term
Term1
Num=1
Z=50 Ohm
Ref
TLOC
TL3
Z=50.0 Ohm
E=137.9
F=5 GHz
TLIN
TL1
Z=50.0 Ohm
E=146.2
F=5 GHz
S2P
SnP
SnP1
File="D:\f341433a.s2p"
TLIN
TL2
Z=50.0 Ohm
E=116.8
F=5 GHz
Ref
TLOC
TL4
Z=50 Ohm
E=159.3
F=5 GHz
Term
Term2
Num=2
Z=50 Ohm

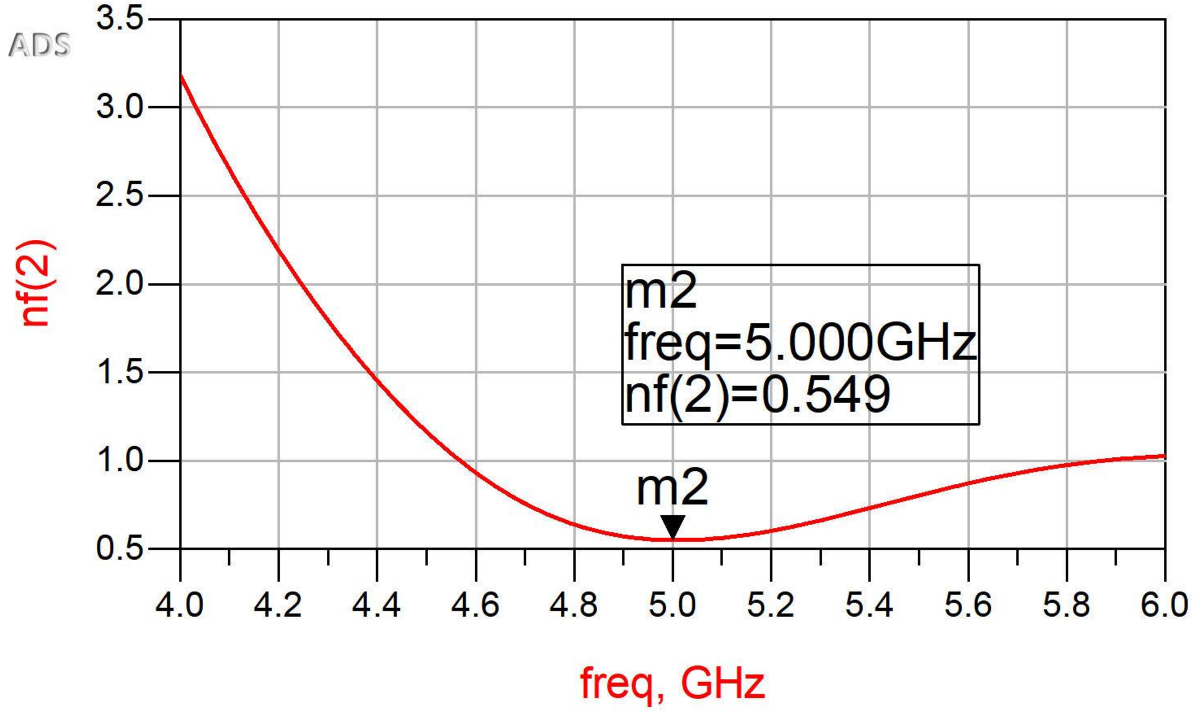


ADS
m1
freq=5.000GHz
dB(S(2,1))=11.475
dB(S(2,1))
freq, GHz

ADS



Term
Term1
Num=1
Z=50 Ohm
Ref
TLOC
TL3
Z=50.0 Ohm
E=137.9
F=5 GHz
TLIN
TL1
Z=50.0 Ohm
E=146.2
F=5 GHz
S2P
SnP
SnP1
File="D:\f341433a.s2p"
TLIN
TL2
Z=50.0 Ohm
E=116.8
F=5 GHz
Ref
TLOC
TL4
Z=50 Ohm
E=159.3
F=5 GHz
Term
Term2
Num=2
Z=50 Ohm

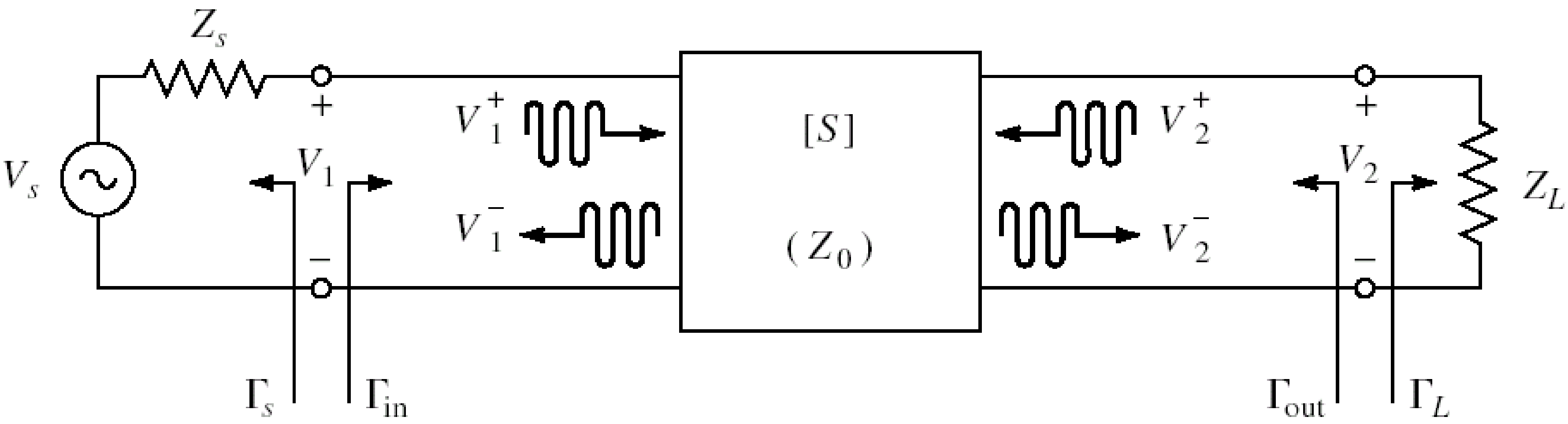


ADS
nf(2)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
m2
freq=5.000GHz
nf(2)=0.549
m2
4.0
4.2
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
freq, GHz

Microwave Amplifiers

Power Gain of Microwave Amplifiers

Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - power gain
  - noise (sometimes – small signals)
  - **linearity** (sometimes – **large signals**)

Bibliografie

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Pozar, Microwave Engineering

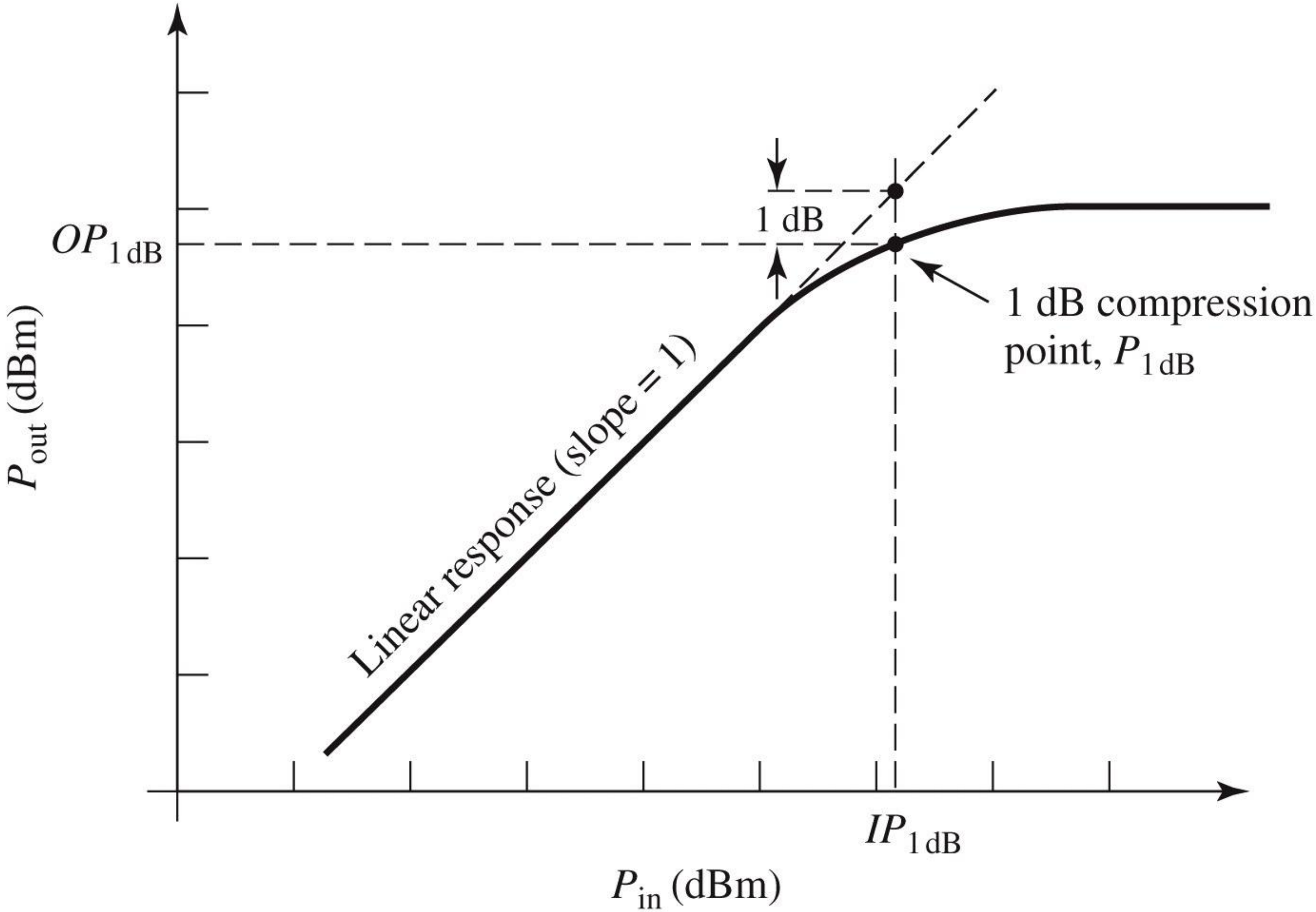

Figure 10.15

Pozar, Microwave Engineering

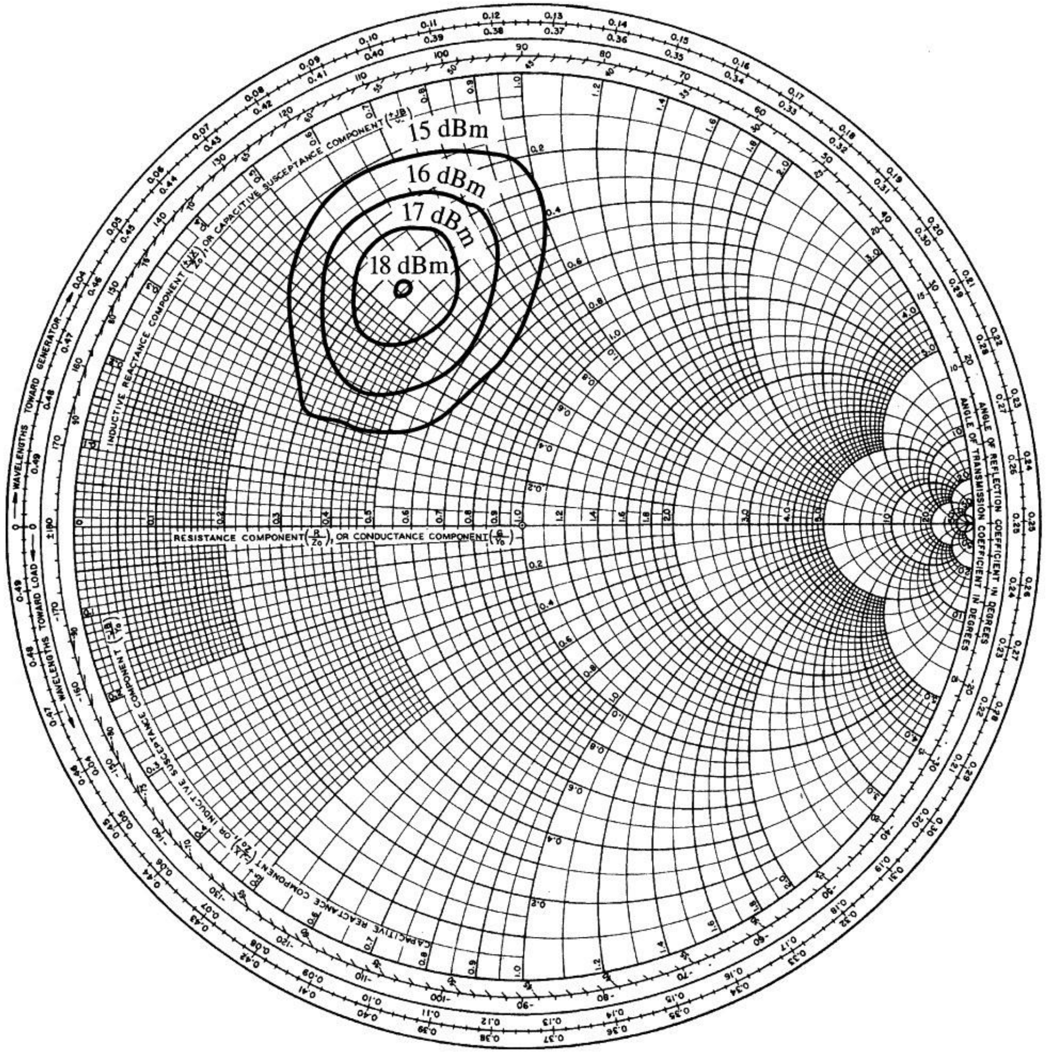

Figure 12.21

Contact

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